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Analysing Spatial and Temporal Variability of Solar Radiation Using Wavelet Spectrum Transforms: Case Studies from Nigeria and the United States

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ABSTRACT

Solar radiation, a fundamental driver of Earth's climate system, exhibits significant spatial and temporal variability across different geographical locations. This study employs wavelet spectrum transforms to analyse hourly solar radiation data spanning two decades over selected cities: Ibadan and Lagos in Nigeria, and New Richmond in the United States. Continuous wavelet transforms with the Morlet wavelet function are utilized to dissect the complex dynamics of solar radiation patterns. The analysis reveals distinct temporal trends and spatial distributions of solar radiation, providing insights into environmental sustainability, agricultural productivity, and socio-economic implications. By elucidating the intricate relationship between solar radiation variability and geographical locations, this study contributes to inform decision-making in renewable energy planning, climate adaptation strategies, and public health interventions.

INTRODUCTION

Solar radiation plays a crucial role in Earth's energy balance, influencing climate dynamics, agricultural productivity, and renewable energy generation (Foufoula-Georgiou & Kumar, 1994). Understanding the spatial and temporal variability of solar radiation is essential for assessing its impact on various aspects of human life and the environment (NATO Science Series, 2022). Therefore, the solar radiation reaching the surface of the Earth can be direct, scattered, absorbed or reflected in the atmosphere. The scattered solar radiation can as well diffuse into the atmosphere and formed a field of radiation. Global solar radiation is the total sum of longwave radiation and shortwave radiation. Therefore, in this context, our study aims to analyse solar radiation patterns over selected cities, namely Ibadan and Lagos in Nigeria, and New Richmond in the United States.

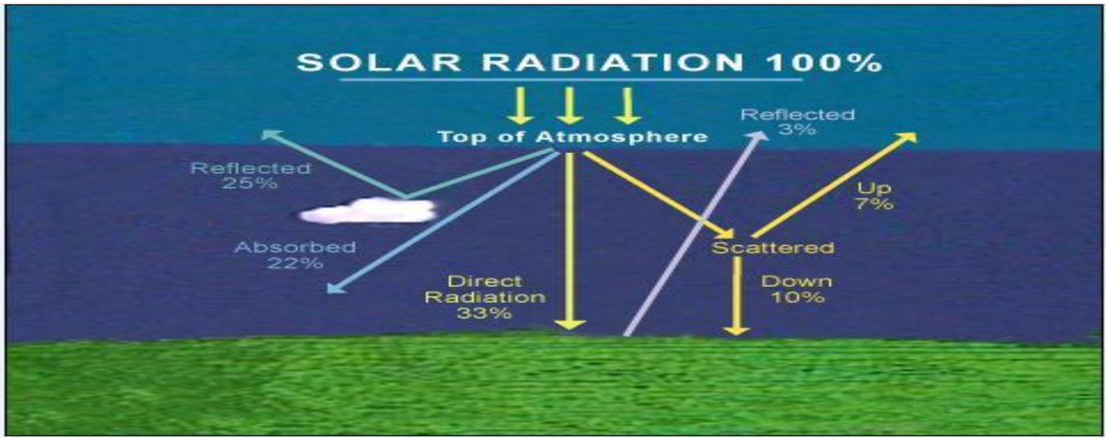

Figure 1: Diagram of Global Radiation (obtained from https://qsstudy.com/wp-content/uploads/2017/12/SOLAR-RADIATION.jpg)

Wavelet Transforms

Wavelet spectrum transforms offer a powerful tool for analysing temporal and spatial variations in solar radiation data (Hong et al., 2017). By decomposing time-series data into frequency-time space, wavelet transforms allow for the identification of dominant modes and their temporal evolution (Torrence & Compo, 1998). This study employs continuous wavelet transforms with the Morlet wavelet function, a widely used approach for analysing non-stationary and non-periodic data (Verwichte, 2017). It is more suitable for the analysis of information of frequency domain, a short cycle of time compared to, a long cycle of time and also possess multi-resolution frequency and time domains (Hong et al, 2017).

Through the analysis, the study aims to elucidate nuanced trends in solar radiation variability, providing insights into environmental sustainability, agricultural practices, and socio-economic implications. By elucidating the intricate relationship between solar radiation variability and geographical locations, this study contributes to informed decision-making in renewable energy planning, climate adaptation strategies, and public health interventions. Whereas, the objectives of the study are to analyse the solar radiation over the selected locations to know how they are affecting the lives of the habitants of the selected locations, and write python code for the continuous wavelet analysis using the Morlet wavelet function at a 5% significant level.

According to Torrence and Compo (1998), wavelet analysis has become a general tool for analysing power variation within a time series. This aids time series decomposition into time-frequency space to control both the variability of dominant modes and the way these modes vary in time. However, wavelet transform has been used for several studies in geophysics such as a tropical convention, El Nino Southern Oscillation (ENSO), atmospheric cold fronts, Central England Temperature, dispersion of ocean wave growth and breaking of coherent structures in turbulence flows (Torrence and Compo, 1998), economic studies – military budget growth and inflation (Yu, 2022), atmospheric physics (Adewole et al., 2020), and others. Furthermore, Fidrmuc et al., (2014) studied the globalization and business cycles in China and G7 countries using continuous wavelet analysis in a research article titled wavelet spectrum analysis of business cycles of China and G7 countries. Whereas, that research work only focused on the economic status of China and other G7 countries. Wavelet spectrum analysis of air temperature and relative humidity in some selected stations in Nigeria was studied by Falayi et al., (2020) to examine the variation of air temperature and relative humidity in eight meteorological stations across Nigeria. They discovered that the relative humidity increases as the air temperature reduce through continuous wavelet analysis. But they could only cover northern parts of Nigeria and focused on the air temperature and relative humidity. Equally, Cantero et al., (2016) analysed the railway bridge response in forced vibration in an article titled time-frequency analysis of railway bridge response in forced vibration, to study the railway bridge responses exited by traversing trains using continuous wavelet transforms with the modified Littlewood-Paley basis. Wang (2022) studied the defence budget growth and inflation from a time-frequency perspective in Great Britain and the US in the article titled defence budget growth and inflation: a wavelet-based study of the US and Great Britain. The research article analysed the finance budget growth using continuous wavelet analysis, whereas Wang (2022) examines the defence budget and inflation in Great Britain and the US.

MATERIALS AND METHODS

Data Collection

Hourly solar radiation data for the cities of Ibadan and Lagos in Nigeria, as well as New Richmond in the United States, were obtained from https://www.meteoblue.com with history + package using Era 5 satellite. The data spanned from January 1st, 2000, to December 31st, 2020, providing a comprehensive dataset for analysis.

Study Locations

The study areas of this research work are badan, the capital of Oyo state and Lagos, the commercial capital of Nigeria, both in the southwestern part of the federal republic of Nigeria, as well as the New Richmond, Clement County, Ohio, United States of America.

Continuous Wavelet Transform CWT

Continuous wavelet transforms were applied to the solar radiation data using the Morlet wavelet function, which is suitable for analysing non-stationary and non-periodic data (Hong et al., 2017). The Morlet wavelet function offers a robust approach for decomposing time-series data into frequency-time space, enabling the identification of dominant modes and their temporal evolution (Torrence & Compo, 1998). Examples of non-stationary signals are audio signals, atmospheric data, meteorological data, electrocardiograms and others. CWT is a common device which augments localised structures of a given shape or periodicity for a given scale while structures with scales far detached (Verwiche, 2017). However, instead of scaly parameters changing, CWT analyses data with a small number of scales with varying translational numbers at every scale. Continuous wavelet analysis comprises tests of wavelet coherence, cross–wavelet transforms, assemblies of routine for wavelet transform, sample scripts and Fast

Fourier Transform, FFT, algorithm with statistical analysis – (Sebastian, 2017). One-dimension and two-dimension are the two functions of CWT with different mother wavelets (Ψ) (Verwiche, 2017).

However, Grinsted et al (2004) and Torrence & Compo (1998) mentioned that the CWT can be expressed as $P_a^X(b)$ which is equal to the translating and scaling parameter function and equal to:

$$P_a^X(b) = \sqrt{\frac{\delta t}{b}} \sum_{n=0}^{N-1} x_{a''} \Psi^* \left[\frac{(a'-a)\delta t}{b}\right] \quad (1)$$

Where a is the translated wavelet across the signal, b is the scaling parameter which is also equal to 1/f, t is the time, $x_{a''}$ is the convolution of x_a, $P_a(b)$ is the function of translating and scaling parameters, and Ψ^* is the conjugate of the mother wavelet.

The continuous wavelet transform CWT must satisfy a condition of admissibility which require the mean to be zero (Kelly et al, 2003). This can be expressed as:

$$\int_R \Psi(t)\, dt = 0 \quad (2)$$

The mother wavelet (Ψ) uses a Gaussian shape to increase its argument absolute value which consists of a damped sinusoidal wave- (Wilks, 2019). According to Torrence and Compo (1998), the function of the non-orthogonal wavelet is applicable in the mother wavelet (Ψ), which is a function of the parameter of non-dimensional time (t) that comprises a Gaussian modulated plane wave. There are different mother wavelets regarding the continuous wavelet transform CWT which are the Haar mother wavelet, Mexican hat mother wavelet, Gaussian mother wavelet and Morlet mother wavelet. Therefore, I shall make use of the Morlet mother wavelet with respect to the scope of this research, because the Morlet mother wavelet is the most suitable mother wavelet for the CWT. This can be expressed as:

$$\Psi_\circ(t) = \pi^{-\frac{1}{4}} e^{i\omega_\circ t} e^{-\frac{t^2}{2}} \quad (3)$$

Where $\omega_\circ$ is the non-dimensional frequency, where $\omega_\circ = 6$, – with respect to this research work and t is dimensionless time. The non-dimensional frequency $w_\circ$ must be constant throughout the analysis of the datasets (Kelly et al, 2003; Torrence and Compo, 1998).

The Scaling Choice

The scaling choice according to Torrence and Compo (1998) is to figure out an appreciative data compression which applies to arbitrary scales in non-orthogonal wavelet analysis. Whereas, an analyst is restricted to a discrete set of scales in orthogonal wavelet analysis. Therefore, it is imperative to select the set of scales s to apply in the wavelet transform of a specified wavelet function which can be conveniently expressed as fraction powers of two:

$$b_j = b_0 2^{j\delta j}, \quad j = 0, 1, 2, \ldots \ldots \ldots \ldots J$$

$$J = \frac{\log_2\left(\frac{N\delta t}{b_0}\right)}{\delta j} \quad (4)$$

Therefore, $J = \frac{\log_2\left(\frac{N}{2}\right)}{\delta j}$

where b_0 is the smallest resolvable scale, j is the determinant of the largest scale, J is the number of scales and N is the number of points. Therefore, from the equations above, the scales of the research work are 331 in total that spanning between 2/8767 (0.00023) years and 330 years, where J is 330, $\delta j = 0.05$, $\delta t = 1/8767$ year, $b_0 = 2\ \delta t$ ($b_0 = 2/8767$ year), and N = 184103.

Normalization

According to Torrence and Compo (1998), in order to normalize a wavelet transform, it is pertinent to note that every scale is directly comparable to another scale and other wavelet transforms of the other time series. These are then weighed not based on the wavelet function but the amplitude of the coefficients of Fourier, x_k. However, every scale wavelet shall be normalized to possess singular energy.

$$\psi(b\omega_k) = \sqrt{\left(\frac{2\pi b}{\delta t}\right)}\, \psi_0(b\omega_k) \quad (5)$$

The non-scaled wavelet transform is defined to have singular energy which is

$$\int_{-\infty}^{\infty} |\psi(\omega')|^2 d\omega' = 1 \quad (6)$$

Therefore, at every scale normalization results to:

$$N = \sum_{k=0}^{N-1} |\psi(b\omega_k)|^2 \quad (7)$$

Where N is the number of points, and ω_k is the angular frequency.

However, with regard to the convolution formula which is implemented in this research work, the normalisation is:

$$\Psi\left[\frac{(a'-a)\delta t}{s}\right] = \sqrt{\left[\frac{\delta t}{s}\right]}\, \psi_0 \left[\frac{(a'-a)\delta t}{s}\right] \quad (8)$$

Where the mother wavelet ($\Psi_\circ(t)$) is normalised to gain singular energy.

Wavelet Power Spectrum Analysis

Wavelet power spectrum can be interpreted as the local phase, which can also be expressed as $|P_n^x(s)|^2$ (Torrence and Compo, 1998; Grinsted et al, 2004) where:

$$P_a^x(b) = \sqrt{\frac{\delta t}{b}} \sum_{a'=1}^{N} x_{a'} \Psi_0 \left[\frac{(a'-a)\delta t}{b}\right] \quad (9)$$

Therefore,

$$|P_a^x(b)|^2 = \left[\left|\sqrt{\frac{\delta t}{b}} \sum_{a'=1}^{N} x_{a'} \Psi_0 \left[\frac{(a'-a)\delta t}{b}\right]\right|\right]^2 \quad (10)$$

Also, Falayi et al (2020) defined the wavelet power spectrum as:

$$P_a^{xx}(b) = \left[|\sqrt{\frac{\delta t}{b}} \sum_{a'=1}^{N} x_{a'} \Psi_0 \left[\frac{(a'-a)\delta t}{b}\right]|\right]^2 \quad (11)$$

The expectation value for $|P_n^x(b)|^2$ is equal to the total number of points multiplied by the expectation value of

$|x_{a'}|$, by comparing the typical wavelet power spectra (Torrence & Compo 1998). This is expressed below:

$$\overline{|P_a^x(b)|^2} = N \times \overline{|x_{a'}|} \quad (12)$$

Meanwhile, the general complexity of the wavelet function Ψ(t) results in the complexity of the wavelet transform $P_a^x(b)$

Statistical Analysis

Descriptive statistics, including mean, standard deviation, minimum, maximum, and percentiles, were computed for the solar radiation data over each location. These statistics provided insights into the variability and distribution of solar radiation patterns across the study period.

Python Programming

The analysis was conducted using the Python programming language in the Spyder environment. Python libraries such as pycwt for wavelet analysis, pandas for data manipulation, NumPy for numerical computations, and matplotlib.pyplot for visualization were utilized.

RESULTS AND DISCUSSION

Statistical Analysis

Descriptive statistics were computed for the hourly solar radiation data collected from Ibadan, Lagos, and New Richmond over the twenty-year study period. Table 1 presents the statistical summary of solar radiation values for each location. The mean solar radiation values were 609.29Wm^{-2} for Lagos, 608.79Wm^{-2} for Ibadan, and 494.28Wm^{-2} for New Richmond. Notably, New Richmond exhibited the lowest mean solar radiation values among the three locations. The table also encompasses count (the total number of the data), mean data, standard deviation, minimum values, maximum values, 25% percentile, 50% percentile (median), and 75% percentile values of Solar Radiation over Ibadan, Lagos, and New Richmond.

Table 1: Statistical Data of Solar Radiation over the Study Locations

	Lagos Solar Radiation	Ibadan Solar Radiation	New Richmond Solar Radiation
Count	184104.00	184104.00	184104.00
Mean	609.29	608.79	494.28
Standard deviation	265.57	270.85	267.16
Minimum value	0.00	0.00	0.00
Maximum value	1407.00	1438.00	1370.00
25%	413.00	413.00	311.00
50%	433.00	438.00	383.00
75%	799.00	796.00	626.00

Discussion of Table of statistical data of solar radiation

Table 1 shows the statistical data of Lagos, Ibadan and New Richmond which contains the total number of 184,104 data set over each of the three study locations, which are the details of the twenty-one (21) year hourly data over Lagos, Ibadan and New Richmond. The mean values Solar Radiation over Lagos, Ibadan and New Richmond respectively are 609.289065Wm^{-2}, 608.791172Wm^{-2} and 494.276469Wm^{-2}. It can be inferred that new Richmond has the lowest values out of all three locations, which implies that less solar energy would be received for energy production and agricultural purposes in the new Richmond compared to the other two study locations. It also shows that Lagos has the highest mean values compared to Ibadan and New Richmond. It can also be deduced from table 1 that the maximum value of Solar Radiation is 1407.000Wm^{-2}, 1438.000Wm^{-2}, and 1370.000Wm^{-2} over Lagos, Ibadan and New Richmond respectively, where Ibadan has the highest value, but these locations have the same minimum values of 0. The values of the standard deviation of data in Lagos, Ibadan and new Richmond respectively are 265.571, 270.851 and 267.157. It may be inferred that southwestern Nigeria or Nigeria entirely has the highest Solar Radiation values compared to the new Richmond or the US as whole, which means that Solar Radiation may cause more harm (Archer, 2008) to the relatively exposed parts of an animal and others living in southwestern Nigeria or Nigeria as a whole compared to the US.

Result of Wavelet Analysis

Wavelet analysis was performed to examine the temporal variability of solar radiation at each location. Figures 2, 3, and 4 depict the wavelet power spectra of solar radiation for Lagos, Ibadan, and New Richmond, respectively. The analysis revealed distinct patterns of solar radiation variability across different temporal scales, with dominant modes identified in each location.

Lagos Wavelet Analysis Results

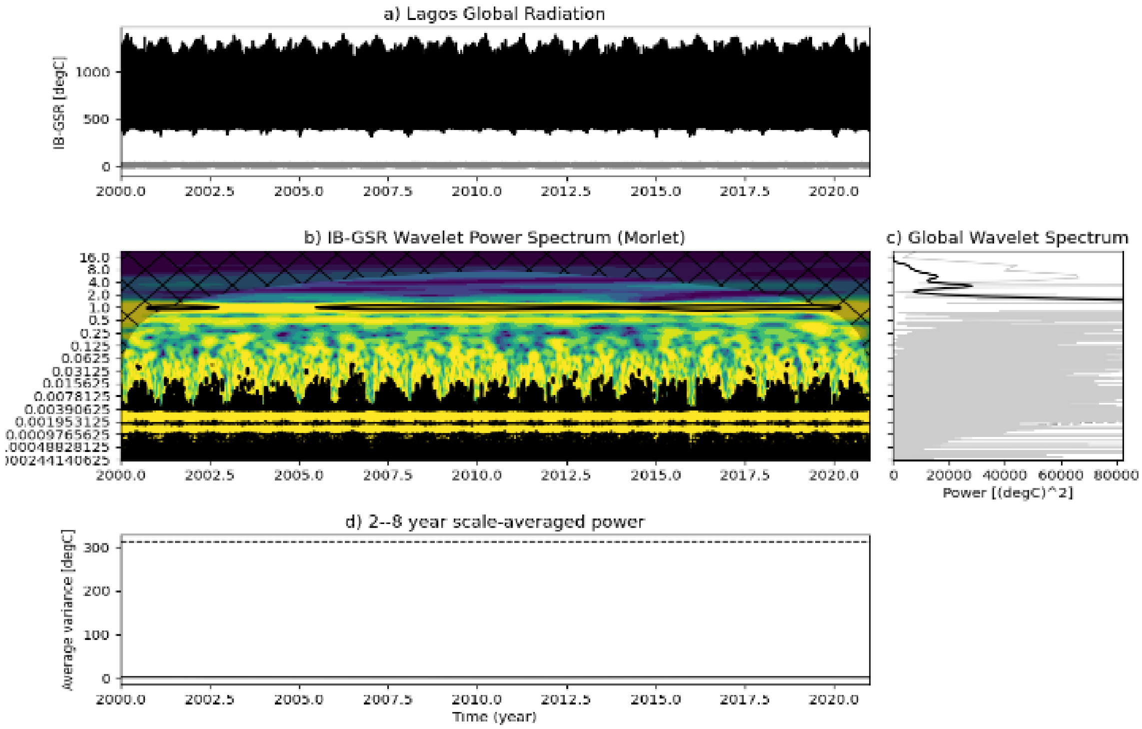

Figure 2: Graph of Lagos SR Wavelet Analysis

Ibadan Wavelet Analysis results

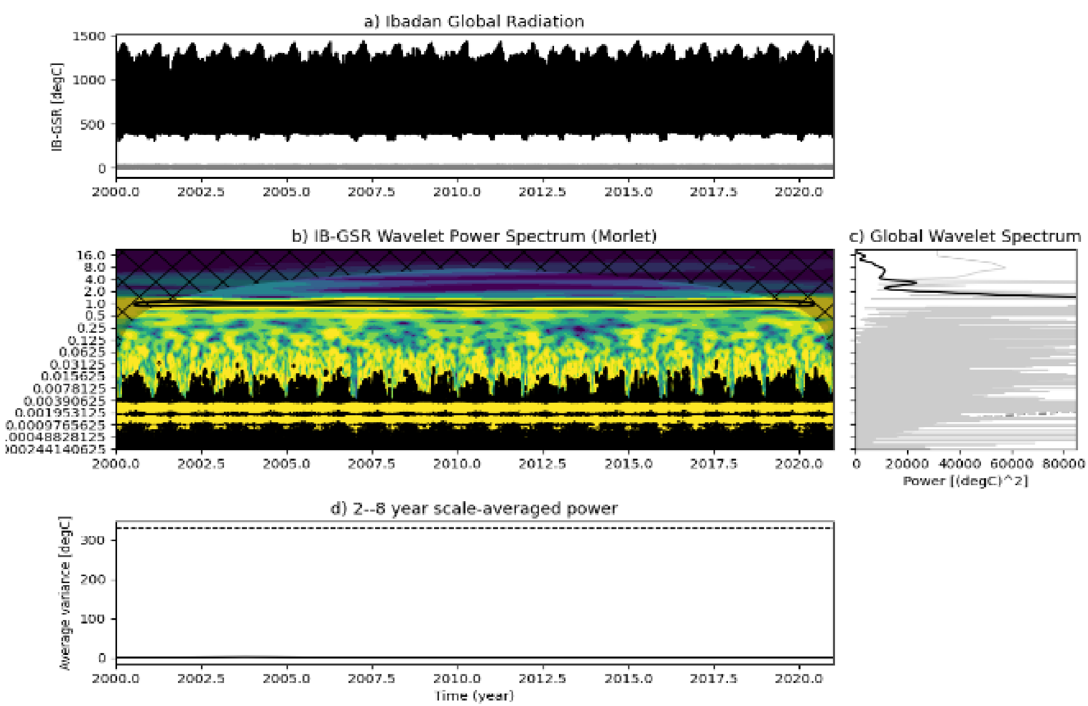

Figure 3: Graph of Ibadan SR Wavelet Analysis

New Richmond Wavelet Analysis

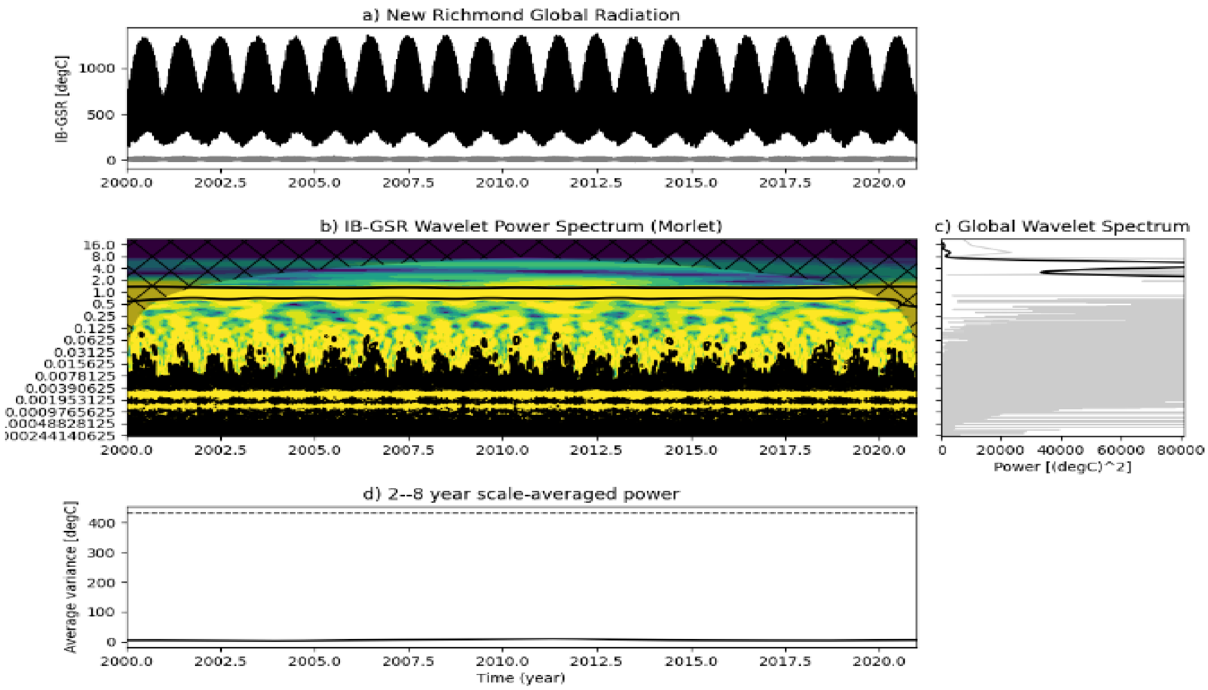

Figure 4: Graph of New Richmond SR Wavelet Analysis

Wavelet analysis provided further insights into the temporal variability of solar radiation, revealing patterns of variability at different time scales. However, Figures 1, 2 and 3 illustrate the Solar Radiation wavelet analysis of Lagos, Ibadan and New Richmond which comprises the time series, Solar Radiation wavelet power spectrum using Morlet function (Morl 6), with dt = 1/8767 per year, δj = 0.05, t0 = 2000, s0 = 2*dt and S = 330, global wavelet spectrum and the scale-average power. In the Figure 1, 2 and 3 show four different plots showing the (a) time series over each location, (b) wavelet power spectrum using Morlet (Morl 6) function with designated colours black, yellow and purple, that signified the occupied space in the plots, the dangerous zone and extremely dangerous zone accordingly, (c) the 2 to 8 year scale average power spectrum which show no difference over the selected locations between the stipulated period of years and the global power spectrum that shows the trends in the data.

Discussion

The findings of this study shed light on the spatial and temporal variability of solar radiation across the selected cities of Ibadan, Lagos, and New Richmond. The statistical analysis revealed notable differences in mean solar radiation values among the study locations, with New Richmond exhibiting the lowest values. This difference may be attributed to various factors, including geographical location, atmospheric conditions, and local climate patterns (Hong et al., 2017). The wavelet analysis further elucidated the temporal variability of solar radiation, revealing distinct patterns of variability at different time scales. In Lagos and Ibadan, for instance, the wavelet power spectra exhibited fluctuations in solar radiation intensity across multiple temporal scales, indicating the presence of both short-term and long-term variability. Conversely, New Richmond showed relatively stable solar radiation patterns, with fewer fluctuations observed over the study period. These findings have important implications for various sectors, including renewable energy planning, agriculture, and public health. In regions with high solar radiation intensity, such as Lagos and Ibadan, there may be opportunities for increased solar energy generation and agricultural productivity. However, careful consideration of factors such as cloud cover, humidity, and air pollution is necessary to maximize the benefits of solar radiation while mitigating potential risks (NATO Science Series, 2022). Therefore, this study contributes to general understanding of solar radiation variability and its impact on different regions. By combining statistical analysis with wavelet analysis, we have gained insights into both the average solar radiation levels and the temporal dynamics of solar radiation variability. Future research could focus on refining predictive models for solar radiation patterns and exploring innovative strategies for harnessing solar energy in diverse geographical contexts.

CONCLUSION

In summary, this study analysed the spatial and temporal variability of solar radiation over the cities of Ibadan and Lagos in Nigeria, as well as New Richmond in the United States, using a combination of statistical and wavelet analysis techniques. The findings revealed significant differences in mean solar radiation values among the study locations, with Nigeria experiencing higher solar radiation levels compared to New Richmond. The statistical analysis provided insights into the average solar radiation levels over the twenty-year study period, with New Richmond exhibiting the lowest mean values. These findings have implications for energy planning and agricultural practices, with regions experiencing higher solar radiation levels potentially benefiting from increased solar energy generation and agricultural productivity. The wavelet analysis further elucidated the temporal variability of solar radiation, revealing distinct patterns of variability at different time scales. While Lagos and Ibadan exhibited fluctuations in solar radiation intensity across multiple temporal scales, New Richmond showed relatively stable solar radiation patterns over the study period. Overall, this study contributes to our understanding of solar radiation variability and its implications for different regions. By combining statistical analysis with wavelet analysis, we have gained insights into both the average solar radiation levels and the temporal dynamics of solar radiation variability. Future research could explore the development of predictive models for solar radiation patterns and investigate innovative strategies for harnessing solar energy in diverse geographical contexts.

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