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# Analytical Study of Viscous Fluid Movement in a Rectangular Pipe using Diffusion Magnetic Resonance Equation

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**ABSTRACT**

Silicene, a two-dimensional material analogous to graphene, has garnered Diffusion Magnetic Resonance Imaging (DMRI) equation is used in this research work to examine the flow of fluid in a rectangular. Having previously considered flow in cylindrical and spherical coordinates, this study explores the rectangular channel of a three dimensional - (3D) flow using DMRI equation evolved and solved analytically using the method of separation of variables (MSV) with appropriate boundary conditions applied. Relaxation times of three viscous fluids were used - crude oil, oil wax and black oil in the simulation and the values of magnetization registered by each fluid recorded. The results obtained showed that oil wax has the highest value of magnetization followed by crude oil and then black oil. The study underscores the multivarious ways diffusion MRI can be applied and its use in the analysis of flow of viscous fluid through different geometrical channels.

**Keywords**:
Relaxation times,
Radio-frequency field,
Magnetic field,
Magnetization.

## INTRODUCTION

Nuclear Magnetic Resonance, NMR, is a powerful analytical technique that exploits the magnetic properties of certain atomic nuclei. When placed in a magnetic field, nuclei with an odd number of protons or neutrons will resonate at a specific frequency when subjected to radiofrequency (RF) pulses. This resonance provides valuable information about the local environment and molecular structure of compounds. One common use of NMR is to figure out the structure of organic compounds and study how molecules move. It is also used to measure number of different nuclei in a sample. Nuclear magnetic resonance and magnetic resonance imaging, MRI, are imaging techniques that use the principles of nuclear magnetic resonance to visualize and evaluate different structures in the body. NMR and MRI are closely related and often used interchangeably.

The concept of NMR and MRI revolves around the behavior of atomic nuclei in a magnetic field. When placed in a magnetic field, the nuclei align themselves with the field. By applying radio waves, the nuclei can be temporarily disturbed from their aligned state. When the radio waves are turned off, the nuclei return to their aligned state, emitting energy that can be detected and used to create an image. NMR and MRI provide valuable information about the structure, function, and composition of tissues and organs. They are noninvasive, do not involve exposure to ionizing radiation.

In their study, Dada *et al.* (2008) showed that nuclear magnetic resonance techniques have been proved to be a powerful and reliable tool in studying flow in restricted geometries because in a few minutes, it accurately provides self-diffusion coefficient for the individual components or multi-component systems. Hence, it is particularly useful for studying diffusion.

MRI is a very powerful tool and has application in various aspects of human life especially in studying the behaviour of fluids generally. It is a method that has been applied severally by scientists and has proved to be very effective in revealing to minute details properties and activities of particles of fluids under consideration.

Several scientists have applied MRI in carrying out a lot of investigation of fluid in the field of medicine, physics, radiology, oil and gas and so on, Awojoyogbe *et al.* (2011). Diffusion magnetic resonance imaging (DMRI) equation has been applied to study and analyse the discontinuities of flow of fluids in a symmetric cylindrical channel. In their work, Yusuf *et al.* (2019a) showed how DMRI equation was used to identify, determine and distinguish between partial and total blockage. Time dependent Bloch NMR flow equation was transformed to diffusion advection equation for the qualitative analysis of nuclear magnetization. The result obtained was used to study fluid flow in blood vessels under different bio-physico-geometrical conditions by Dada *et al.* (2010). MRI has also been further used to image the nature of the materials that can cause obstructions or blockage of fluids in a cylindrical pipe, Yusuf *et al.* (2019b).

The analytical solutions of the transient solid-state diffusion of the single-phase and two-phase in radial spherical and cylindrical geometries have also been considered. The modified differential equations were solved using error function method and obtained solutions used to analyse the diffusion interface position as a function of time and position in spheres and cylinders. The analytical solutions were validated with the results of a numerical approach called enthalpy method. The model was proved to be general, as far as, a semi-infinite solution for diffusion problems with phase change exist in the form of error function that enables them to derive the exact closed-form of analytical solutions by updating the root at each radial position of the moving boundary interface, Ferreira *et al.* (2021).

Analytical solution of magnetic resonance imaging (MRI) equation was used to study and analyse the general behaviour of fluid flow in human living tissues which, by assumption, was considered to be cylindrical, Yusuf *et al.* (2010). Another useful application of MRI was carried out with $T_1$ and $T_2$ relaxation times from Bloch equations. These relaxation times were used to estimate the age of human organs, Olaoye *et al.* (2021).

Datta and Pal (2018) studied one dimensional radial diffusion equation in spherical coordinate system using the Lattice Boltzmann scheme. The scheme was investigated and there was a great analogy between the simulation and the analytical solution. The result obtained shows that the scheme will be able to simulate the radial diffusion equation accurately.

Gharehbaghi (2017) developed a numerical model based on the Finite Volume Method to predict a time dependent one-dimensional advection diffusion equation with variable coefficient in a semi-finite domain. The third and fourth order schemes were used to solve the governing equations. Two dispersion problems were used to simulate various conditions as first solute dispersion along steady flow through inhomogeneous domain and secondly solute dispersion along temporarily dependent unsteady flow through inhomogeneous domain. Then, the results of third-order and fourth-order Finite Volume Method (FVM) were more accurate than the result of quick scheme FVM and among the three approaches, the fourth order FVM was achieved to present the best predictions.

In another research, a simple and fast technique for solving the time dependent Bloch equations by using matrix operation method was derived by Murase and Tanki (2010). This method was validated in case of constant radio-frequency irradiation by comparing with the analytical solutions which indicates a good agreement between the methods.

Singh and Srivastava (2020) presented a numerical simulation of fractional-order and integer-order Bloch equations that occurs in NMR by using the Jacobi-polynomials. The numerical solution of the technique varies consistently at distinct values of fractional-order time derivatives and integer-order solutions of the technique were identical to the exact solution of the Bloch equations.

Belyaev *et al.* (2015) investigated two dimensional variables of fluid particles of motion in a curved duct using numerical analysis. Navier-Stoke's equation was used to model the phenomenon. Control Volume (CV) approach was used to discretize the initial equations. The result discovered shows that the trajectory of the moisture reduces the motion and its speed.

In the work of Fatumbi and Fenuga (2018), reference was made to micropolar fluids which represent fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium, where particles deformation is ignored. These are group of fluids with non-symmetric stress tensor that are called polar fluids which constitute a substantial generalization of the Navier-Stoke's model. These fluids offer a mathematical model for investigating the flow of complex and complicated fluids such as suspension solution, animal blood, liquid crystals, polymeric fluids and clouds with dust.

Similarly, Isede *et al.* (2023) pioneered the classical unidirectional laminar flow problem of an incompressible and viscous electrically conductive fluid permeated by a non-varying magnetic field; applied transversely to the parallel walls of the channel. Popoola *et al.* (2016) examined the two-dimensional steady flow of heat and mass transfer in an incompressible magneto hydrodynamic viscous-elastic fluid pass a stretching sheet in the presence of thermal diffusion and chemical reaction. The similarity transformation method was used to convert the partial differential equations governing the flow of heat and mass transfer properties into the coupled ordinary differential equations.

Mallin *et al.* (2011) and Olaide *et al.* (2020) also applied NMR pulse to determine the properties of glycerin and mineral oil. The application of nuclear magnetic resonance measurement in petroleum industry gives better understanding of the interaction between fluids in the reservoirs and rock properties and one of the best tools for quantifying fluid properties, reservoir properties as well as determining reservoir productivity.

Most of the researches undertaken by the scientists were on cylindrical or spherical regions or shapes. Therefore, it is necessary to consider a situation where fluid flows in a rectangular duct or channel and this is the motivation for this study.

## MATERIALS AND METHODS

### Mathematical Formation

Assuming there is fluid flowing in a rectangular pipe with dimensions $x, y, z$ and applying magnetic resonance to the flow in the pipe, the model equation is depicted as follows (Awojoyogbe, 2004):

:

$$M_{tt} = \frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} + \frac{\partial^2 M}{\partial z^2} + w_c(t) \quad (1)$$

where

$$w_c(t) = \int_0^{t_0} \frac{F_o}{T_o} \gamma B_1(t)\, dt \,. \quad (2)$$

$M$ is the magnetization, $x$ is $x$ – axis; $y$ is $y$ – axis; $z$ is the direction of flow; $t$ is the time and $\gamma$ = gyro-magnetic ratio of fluid spins while $B_1(t) =$ radio-frequency (RF) magnetic field

$$M_{tt} = \frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} + \frac{\partial^2 M}{\partial z^2} + \int_0^{t_0} \frac{F_o}{T_o} \gamma B_1(t)\, dt \quad (3)$$

Note the last function on the righthand side $\int_0^{t_0} \frac{F_o}{T_o} \gamma B_1(t)\, dt = \int_0^{t_0} w_c(\dot{t})\, dt$ is the radio-frequency field applied to perturb the molecules of the fluid, in this case, oil. It is kept aside for the time being while solving the remaining function on the righthand side. Also, since the *flow is assumed to be constant in the direction of z – axis*, therefore (3) reduces to:

$$M_{tt} = \frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} \quad (4)$$

with initial and boundary conditions given as,

$$M(x, y, t) = 0$$
$$M(x, y, 0) = f(x, y)$$
$$M_{tt}(x, y, 0) = g(x, y) \quad (5)$$

Let

$$M = XYT \quad (6)$$

Such that

$$M_{tt} = XYT'', \qquad \frac{\partial^2 M}{\partial x^2} = X''YT \quad \text{and} \quad \frac{\partial^2 M}{\partial y^2} = XY''T$$

Substituting into (4) gives

$$XYT'' = X''YT + XY''T \quad (7)$$

Divide through by $XYT$ ,

$$\frac{XYT''}{XYT} = \frac{X''YT}{XYT} + \frac{XY''T}{XYT} \quad (8)$$

Implying

$$\frac{T''}{T} = \frac{X''}{X} + \frac{Y''}{Y} \quad (9)$$

Equating to an arbitrary value gives

$$\frac{T''}{T} = \frac{X''}{X} + \frac{Y''}{Y} = -\mu \quad (10)$$

Solving

$$\frac{T''}{T} = -\mu \quad (11)$$

$$\Rightarrow \quad T'' + \mu T = 0 \quad (12)$$

Assume

$$T = e^{mt},\ T' = me^{mt},\ T'' = m^2 e^{mt} \quad (13)$$

Equation (12) becomes

$$(m^2 + \mu)e^{mt} = 0 \quad (14)$$

with solution

$$T(t) = ae^{\sqrt{\mu t}} + be^{-\sqrt{\mu t}} \quad (15)$$

Therefore,

$$T(t) = A_{mn} cos\sqrt{\mu_{mn}}t + B_{mn} sin\sqrt{\mu_{mn}}t \quad (16)$$

Similarly, from (10),

$$\Rightarrow \frac{Y''}{Y} + \mu = -\frac{X''}{X} \quad (17)$$

Equating to an arbitrary value gives

$$\frac{Y''}{Y} + \mu = -\frac{X''}{X} = \tau \quad (18)$$

where

$$-\frac{X''}{X} = \tau \quad \Rightarrow -X'' = \mu X \quad or \quad X'' + \mu X = 0 \quad (19)$$

Let

$$X = e^{mx},\ X' = me^{mx},\ X'' = m^2 e^{mx}\} \quad (20)$$

Substituting (20) into (19) gives,

$$m^2 e^{mx} + \tau e^{mx} = 0, \quad (21)$$

$$(m^2 + \tau)e^{mx} = 0 \quad (22)$$

with solution $\quad X = ce^{i.\sqrt{\tau}x} + de^{-i.\sqrt{\tau}x} \quad (23)$

Therefore,

$$X(x) = C_m cos\sqrt{\tau_m}x + D_m sin\sqrt{\tau_m}x \quad (24)$$

From the boundary conditions,

$$X(0) = 0 \quad (25)$$

$$\Rightarrow C_m = 0 \quad (26)$$

Giving $\quad X(x) = D_m sin\sqrt{\tau_m}x \quad (27)$

Also,

$$X(x_o) = 0 \quad (28)$$

Hence from (27)

$$X(x_o) = D_m sin\sqrt{\tau_m}x_o = 0 \quad (29)$$

but

$$D_m \neq 0 \quad (30)$$

therefore

$$sin\sqrt{\tau_m}x = 0 \quad (31)$$

implying that

$$\tau_m = (\frac{m\pi}{x_o})^2 \quad (32)$$

Substituting (32) into (27) gives

$$X(x) = D_m sin\left(\frac{m\pi}{x_o}\right)x \quad (33)$$

Again, from (18),

$$\frac{Y''}{Y} + \mu = \tau \quad (34)$$

$$\frac{Y''}{Y} + \mu - \tau = 0 \quad (35)$$

Assume

$$v = \mu - \tau \quad (36)$$

$$\frac{Y''}{Y} + v = 0 \quad (37)$$

$$Y'' + vY = 0 \quad (38)$$

Let

$$Y = e^{my},\ Y' = me^{my},\ Y'' = m^2 e^{my} \quad (39)$$

Equation (38) becomes

$$m^2 e^{my} + ve^{my} = 0 \quad (40)$$

$$(m^2 + v)e^{my} = 0 \quad (41)$$

Giving

$$Y = ge^{i.\sqrt{v}y} + fe^{-i.\sqrt{v}y} \quad (42)$$

with solution

$$Y(y) = E_n\, cos\sqrt{v}y + F_n\, sin\sqrt{v}y \quad (43)$$

Applying the boundary conditions,

$$Y_n(0) = Y_n(y) = 0 \quad (44)$$

$$Y_n(0) = E_n(1) + F_n(0) = 0 \quad (45)$$

$$E_n = 0 \quad (46)$$

Hence, (43) becomes

$$Y_n(y) = F_n\, sin\sqrt{v}y \quad (47)$$

and $Y_n(y_o) = F_n\, sin\sqrt{v}y_o = 0$ (48)

but $F_n \neq 0$ (49)

$$sin\sqrt{v}y_o = 0 = \sin n\pi \quad (50)$$

implying that

$$v = (\frac{n\pi}{y_o})^2 \quad (51)$$

Substituting (51) into (47) gives

$$Y_n(y) = F_n\, sin\left(\frac{n\pi}{y_o}\right)y \quad (52)$$

Therefore, from (17), (33) and (52),

$$M(x,y,t) = X_m(x)Y_n(y)T_{mn}(t) \quad (53)$$

$$M(x,y,t) = (D_m sin\left(\frac{m\pi}{x_o}\right)x)(F_n\, sin\left(\frac{n\pi}{y_o}\right)y(A_{mn}cos\sqrt{\mu_{mn}}t + B_{mn}sin\sqrt{\mu_{mn}}t) \quad (54)$$

$$M(x,y,t) = (sin\left(\frac{m\pi}{x_o}\right)x)(sin\left(\frac{n\pi}{y_o}\right)y(\alpha_{mn}cos\sqrt{\mu_{mn}}t + \beta_{mn}sin\sqrt{\mu_{mn}}t) \quad (55)$$

where

$$\alpha_{mn} = A_{mn}D_mF_n \text{ and } \beta_{mn} = B_{mn}D_mF_n \quad (56)$$

From (36),

$$\tau + v = \mu \quad (57)$$

Recall that from (32) and (51)

$$\tau_m = (\frac{m\pi}{x_o})^2 \text{ and } v = (\frac{n\pi}{y_o})^2 \text{ respectively} \quad (58)$$

Hence, (57) becomes

$$\mu = (\frac{m\pi}{x_o})^2 + (\frac{n\pi}{y_o})^2 \quad (59)$$

$$\mu = \pi^2(\frac{m^2}{x_o^2} + \frac{n^2}{y_o^2}) \quad (60)$$

while

$$\sqrt{\mu_m} = {}^{\pi}\sqrt{\frac{m^2}{x_o^2} + \frac{n^2}{y_o^2}} \quad (61)$$

Substituting (61) into (55) gives the final solution:

$$M(x,y,t) = \{[\, sin\left(\frac{m\pi}{x_o}\right)x][sin\left(\frac{n\pi}{y_o}\right)y][\, \alpha_{mn}\cos\left(\pi t\sqrt{\frac{m^2}{x_o^2} + \frac{n^2}{y_o^2}}\right) + \beta_{mn}\sin\left(\pi t\sqrt{\frac{m^2}{x_o^2} + \frac{n^2}{y_o^2}}\right)]\} \quad (62)$$

**The Radio Frequency *(RF)* Field**

The Radio Frequency $(RF)$ is usually used to transmit energy into the fluid under consideration in the rectangular channel. This is done to activate the nuclei so as to emit a signal. The $RF$ field is $B_1(t)$ where $B_1(t) = bB_1(t)cos\omega t$. The field is said to be linearly polarized because it oscillates in a single direction $\omega$ is called the irradiation frequency, it is also the reference frequency of the $RF$ transmitter and the detection system, $\omega$ has a value of $\omega = 1.0x10^8 rads^{-1}$ (waldo & Arnold, 1983). Therefore, applying this to the radio frequency set aside from (3), it implies

$$w_c(t) = \int_0^{t_0} \frac{F_o}{T_o}\gamma \boldsymbol{B_1(t)}\, dt\,. \quad (63)$$

with

$$F_o = \frac{M_o}{T_1} \quad \text{and} \quad T_o = \frac{1}{T_1} + \frac{1}{T_2} = \frac{T_1+T_2}{T_1T_2} \quad (64)$$

where

$T_1$ = Longitudinal or spin lattice relaxation time

$T_2$ = Transverse or spin-spin relaxation time

$$\boldsymbol{B_1(t)} = bB_1(t)\cos\omega t \quad (65)$$

$$w_c(t) = \int_0^{t_0} \frac{F_o}{T_o}\gamma bB_1(t)\cos(\omega t)\, dt \quad (66)$$

$$\int_0^{t_0} \frac{bF_o}{T_o}\gamma\cos(\omega t)\, dt = \frac{bF_o}{\omega T_o}\gamma sin(\omega t) \quad (67)$$

Combining with the solution obtained in (62);

$$M(x,y,t) = \{[\, sin\left(\frac{m\pi}{x_o}\right)x][sin\left(\frac{n\pi}{y_o}\right)y][\, \alpha_{mn}\cos\left(\pi t\sqrt{\frac{m^2}{x_o^2} + \frac{n^2}{y_o^2}}\right) + \beta_{mn}\sin\left(\pi t\sqrt{\frac{m^2}{x_o^2} + \frac{n^2}{y_o^2}}\right) + \frac{bF_o}{\omega T_o}\gamma sin(\omega t)]\} \quad (68)$$

## RESULTS AND DISCUSSION

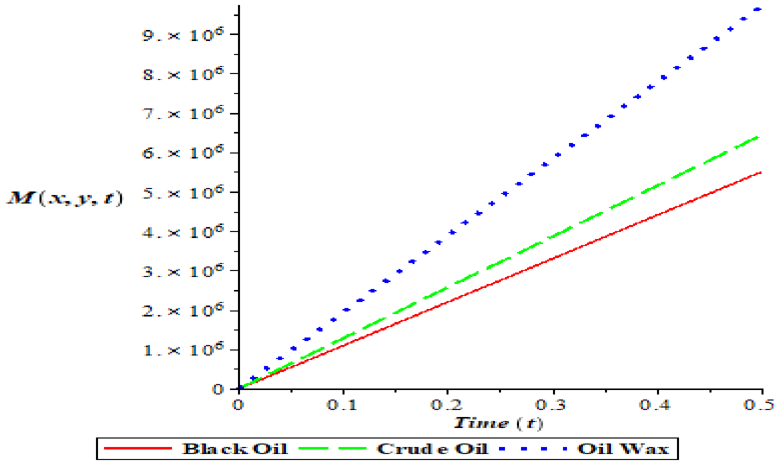


Figure 1: Comparison of black oil, crude oil and oil wax

**Discussion of Results**

From Figure 1, the value of magnetization recorded by the black oil is the least of the three fluids under consideration with a magnetization value of 5 x $10^6$. The crude oil is next to the black oil with a magnetization value of 6 x $10^6$ while the oil wax recorded the largest value of magnetization at 10 x $10^6$. This result agrees with Dada *et al.* (2008) which showed that nuclear magnetic resonance techniques have been proved to be a powerful and reliable tool in studying flow in restricted geometries – spheres and cylinders, as it accurately provides self-diffusion coefficient for the individual components or multi-component systems. The research has shown clearly that diffusion MRI can be used to study the distinguishing features of the minute particles of fluids. This is clearly established from the various applications of MRI as enunciated in the literature review.

It has could be seen that the flow and behaviours of the three fluids considered in this research could be readily detected in the rectangular channels through the application of diffusion magnetic resonance imaging equation. In addition, the different values of magnetization also indicate their different viscosities. Hence, it can be deduced that oil wax is the most viscous, then crude oil and thereafter black oil. This also shows a good agreement with the method used in the analysis of diffusion interface position as a function of time and position in spheres and cylinders which was carried out by Ferreira *et al.* (2021).

## CONCLUSION

The mathematical analysis showing the study of flow of fluids in a rectangular duct using Diffusion Magnetic Resonance Imaging Equation has been carried out. The partial differential equations governing the phenomenon were evolved from the fundamental Bloch equations and solved analytically using the method of separation of variables and appropriate boundary conditions were applied. The fluids used for simulation were crude oil, oil wax and black oil. The result shows that magnetization values of the fluids differ with oil wax having the highest value followed by the crude oil and lastly the black oil. This study shows the effectiveness of diffusion magnetic resonance imaging equations in the study and analysis of flow of fluid in a rectangular channel.

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