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Jerk Chaotic System: Analysis, Circuit Simulation, Control and Synchronization with Application to Secure Communication

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ABSTRACT
Dynamical behaviors of 3D Jerk system were examined in terms of equilibrium, stability, dissipative, and phase space attractor in this study. The system's practical applications were demonstrated through circuit realization and synchronization scheme via active backstepping control with its effectiveness demonstrated in secure communication. The viability of the theoretical model of 3D Jerk system was confirmed using electronic circuit workbench designed in MultiSIM enviroment. A nonlinear feedback controller was designed using the recursive backstepping technique to control and track a desire function. For secure communication application, active backstepping method was adopted to synchronize two identical chaotic systems evolving from different initial conditions. It was demonstrated that when the controller was activated, the systems synchronize successfully. The results of the active backstepping designed controllers were numerically applied in the area of secure communication, with the variable of the drive being encrypted information transmitted through a coupling channel. Using an additive encryption masking scheme, the encrypted signal was a superposition of sinusoidal information specified by period function and chaotic carrier generated from a variable of the Jerk system. The transmitted information signal was successfully retrieved from the chaotic response signal using an inverse function decryption algorithm, thereby confirming the effectiveness and robustness of the designed controller.

Keywords:
Chaos,
Control,
Synchronization,
Secure communication.

INTRODUCTION
Chaotic dynamics in nonlinear science is characterized by unpredictability, irregularity, sensitive to initial conditions and parameters variation (Otti, 2002; Liu & Zhu, 2008).

Recently, nonlinear chaos has becomes a hot topic of research among the scientists, engineers and economists as a result of it application in secure communication, signal processing, random generator, power converter, high performance circuit design for telecommunication (Chua, 1998; Chua & Roska, 2002; Fortuna, Franca & Xibilia, 2009), biomedical engineering application, physical, chemical, biological (Strogatz, 1994) and financial systems (Kyrtsou & Labys, 2006; Kyrtsou & Labys, 2007; Idowu *et al.*, 2018).

Since chaotic dynamics is unpredictable as well as sensitive to initial conditions, the idea to control and synchronizes the chaotic systems has been considered illogical. The year 1990 recorded a major breakthrough in the literature of the chaos dynamics through the pioneering work of Pecora and Carroll (1990) on the chaos synchronization. This great breakthrough revealed the potential application of chaos synchronization in various physical and real world systems.

Chaotic phenomena could be beneficial in some applications and undesirable in many systems, such as; engineering, economic activities and many other physical systems. Therefore the ability to control or transform chaotic behavior is highly of advantage to several fields to improve the systems performance.

Chaos control is to stabilize a known unstable periodic orbit at the equilibrium point or to track any defined function $r(t)$ at any chosen position where as synchronization on the other hand is the linking or coupling of one system trajectories to the corresponding trajectories of the other system such that both systems remains in step (cooperate) with each other throughout the transmission of signal (Njah, 2009; Onma *et al.*, 2014; Onma *et al.*, 2017a).

Numerous methods have been developed and employed to achieved chaos control and synchronization. Among the methods are; active control (Bai & Longngren, (1997); Sarasu & Sandarapandian, 2011a; Sarasu & Sandarapandian, 2011b; Vincent, 2008), adaptive control (Yassen, 2003; Wang & Wang, 2011); El-Dessoky & Yessen, 2012; Onma *et al.*, 2016; Tirandaz & Hajipour, 2017; Idowu, 2019; Onma *et al.*, 2021), backstepping technique (Mascolo, 1997; Laoye *et al.*, 2009; Njah, 2009; Olusola *et al.*, 2011; Onma *et al.*, 2014; Onma *et al.*, 2017a), sliding mode control (Medhafar et al., 2019; Rajagopal, 2017).

In engineering community, great attention has drawn on circuit realization of a chaotic system. Circuit realization is important in implementation of the system in real-world application in chaos scheme communication technologies and information systems (Kemih *et al.*, 2015; Pham *et al.*, 2015; Idowu *et al.*, 2018; Vaidyanathan *et al.*, 2018; Sambas *et al.*, 2019; Adelaja *et al.*, 2021). As a result, this paper analyses and presents the circuit realization of Jerk 3D system.

Secure communication is one of the major motivating factors of chaos synchronization application. The principle of chaos application in secure communication scheme is by using a chaotic oscillator as a broadband signal generation. The chaotic signal is mixed with the information signal to produce unpredictable signal which is transmitted from the drive to the response (Li *et al.*, 2005; Adelakun *et al.*, 2014; Onma *et al.*, 2017a; Onma *et al.*, 2017b).

Synchronous chaotic communication technology is grouped into: chaotic masking technology, chaotic parameter modulation technology and chaotic keying technology. Chaos masking technology is an analog communication (Adelakun *et al.*, 2014; Onma *et al.*, 2017a; Onma *et al.*, 2017b), chaos parameter modulation and chaos keying technology are digital communication technology (Yau *et al.*, 2012; Laoye *et al.*, 2021, Adelaja *et al.*, 2021). This research work gives the details description of communication scheme via active backstepping synchronization in analog communication system.

In light with various applications of chaotic systems; the investigation, control and synchronization behavior of Jerk chaotic system is of great importance. The present work is aimed on; the qualitative properties of Jerk system such as stability, dissipation, equilibra, unstable, attractor, time evolution, circuit designed, tracking control, active backstepping nonlinear control feedback function that able to synchronizing two identical chaotic Jerk systems evolving from different initial conditions and its application to secure communication. The synchronization based on this technique is simple, robust and easy to implement in real applications in secure communication.

MATERIALS AND METHODS

Model Description

The nonlinear system considered here is Jerk 3D chaotic system which is the set of ordinary differential equations defined in equation (1).

$$\dot{x} = y$$
$$\dot{y} = z$$
$$\dot{z} = -x - y - az - bx^2 \qquad (1)$$

Equation (1) is a three-dimensional autonomous nonlinear Jerk system with one quadratic term and two positive constant parameters a and $b$while x, y and $z \in R^n$are the state variables of the system.

Methods

Numerical method and experimental study (electronic implementation) were adopted for the study of dynamics of Jerk system described by equation (1). The electronic implementation was carried out to validate the theoretical/numerical analysis. The dynamics of the Jerk system as stated in equation (1) was analyzed through graphical representation of the system evolution in the phase space, that is, the orbits in the space of the system variables. The Fourth- Order Runge-kutta algorithm was adopted to perform the numerical integration of the model as described in equation (1) in order to study the phase plane of the model and also to design the controllers in order to achieve synchronization of models using Active Backstepping Scheme. Electronic implementation of Jerk system was established using simple OP-AMP's and other appropriate electronic components such as resistors, capacitors and operational amplifiers. The system (1) is designed using Multisim program.

RESULTS AND DISCUSSION

Numerical results, experimental realization of dynamical analysis of Jerk 3D chaotic system are presented. The experimental analysis was simulated on Multisim (workbench) and thereafter, correlation between numerical and experimental results was further considered.

Phase Portraits Analysis

The plots of attractors are presented to show the chaotic nature of the Jerk model. Setting parameters $a = 0.5$ and $b = 0.125$ for system (1) exhibits complex chaotic behavior attractor displayed in Figure 1.

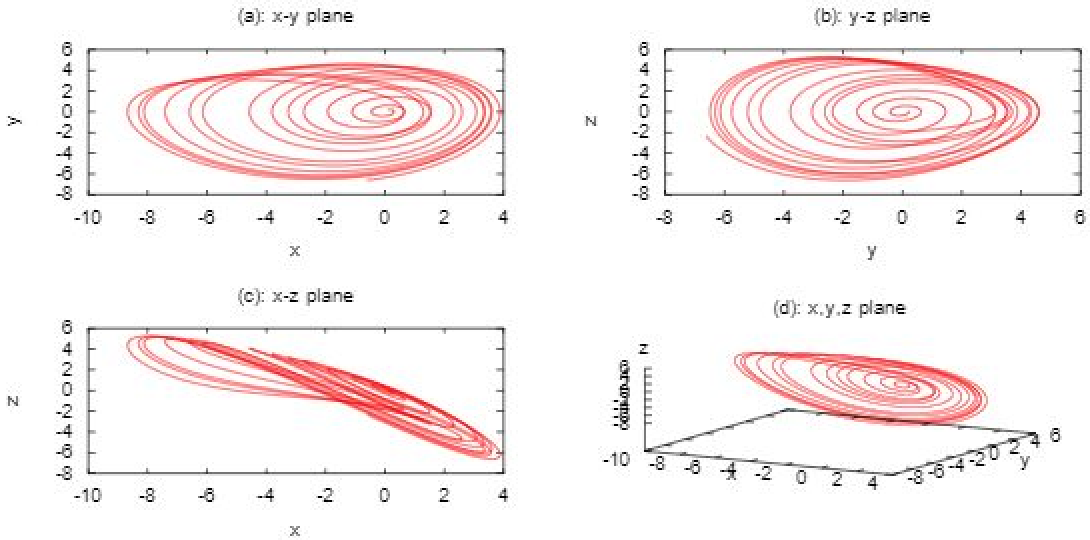


Figure 1: Phase space attractor of Jerk chaotic system with parameter values $a = 0.5$and $b = 0.125$

Dynamical analysis of Jerk 3D chaotic system

Dissipative

The divergence of the Jerk 3D chaotic system (1) could be obtained from equation (2) below:

$$\nabla V = \frac{\partial \dot{x}_1}{\partial x_1} + \frac{\partial \dot{x}_2}{\partial x_2} + \frac{\partial \dot{x}_3}{\partial x_3} \quad (2)$$

$\nabla V = -(a) = -0.5$

According to the range of the system parameter, $-0.5 < 0$. Thus Jerk 3D chaotic system (1) is a dissipative system. All the orbit of this system converges to a specific subset of zero as $t \rightarrow \infty$ exponentially. Hence,$\dot{V}(t) = V_0 e^{-(0.5)t}$, which means for an initial volume V_0, the volume will become $V_0 e^{-(0.5)t}$ at instant t through the flow by the system (1). Thus, there exists an attractor in system (1) as shown in Figure 1 above.

Equilibrium and Stability

The equilibrium of system (1) satisfies these;

$y = 0$

$z = 0$

$$-x - y - az - bx^2 = 0 \quad (3)$$

By linearizing equation (3) at the equilibrium point $E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ for all the parameter values, the Jacobian matrix is given as:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -0.5 \end{bmatrix} = J_{(0,0,0)} \quad (4)$$

In order to compute the eigenvalues of the equation (4), equation (5) is introduced as follow;

$$|\lambda I \quad - \quad J_0| = 0 \quad (5)$$

Hence, from equation (5), equation (4) is transformed as shown in equation (6).

$$\begin{bmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ -1 & -1 & (-0.5 - \lambda) \end{bmatrix} = 0 \quad (6)$$

The characteristics equation of Jacobian system (6) is given in equation (7).

$$-(\lambda^3 + 0.5\lambda^2 + \lambda + 1) = 0 \quad (7)$$

The eigenvalues of equation (6) at equilibrium point $E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$are calculated using MATLAB as:

$\boldsymbol{\lambda_1 = -0.80376, \lambda_{2,3} = 0.15188 \pm 1.10503i}$

In a continuous nonlinear dynamical system, the condition for stability is that all the eigenvalues and Lyapunov exponents must be negative.

Obviously,

$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is a trivial equilibrium.

When$a = 0.5$and$b = 0.125$, the eigenvalues of equilibrium $E_0(0,0,0)$ are$\lambda_1 = -0.80376$, $\lambda_2 = 0.15188 + 1.10503i$and $\lambda_3 = 0.15188 - 1.10503i$.

Then,

$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is a two-dimensional unstable saddle point.

On any initial condition on the negative vector (λ_1), the orbit will converges to the equilibrium point $E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ through the eigen-plane of this negative eigenvector (λ_1) but any deviation along λ_2 and λ_3 (positive eigenvectors) will cause expansion and the orbit will become unstable result to a saddle point. Physically, these results bear that the Jerk system (1) can oscillate chaotically and prohibit the existence of stable fixed point in the system.

Electronic Circuit Design

Here, the electronic circuit is designed to implement Jerk 3D chaotic system (1). The state variables x, y and $z$of system (1) are scaled in order capture the attractors in the dynamical range of operational amplifiers. MultiSIM 14 software is used to design the circuitry shown in Figure 2.

The circuit comprises of three channels to realize the integration addition and substation of the state variables x, y and$z$. The electronic components are resistors, capacitors and operational amplifiers. Using Kirchhoff's laws on the designed circuit, we have equation (8) below.

$$\frac{dVc_1}{dt} = \frac{1}{C_1R_1}Vc_2$$
$$\frac{dVc_2}{dt} = \frac{1}{C_2R_2}Vc_3$$
$$\frac{dVc_3}{dt} = -\frac{1}{C_3R_3}Vc_1 - \frac{1}{C_3R_4}Vc_2 - \frac{1}{C_3R_5}Vc_3 - \frac{1}{C_3R_6}Vc_1^2 \quad (8)$$

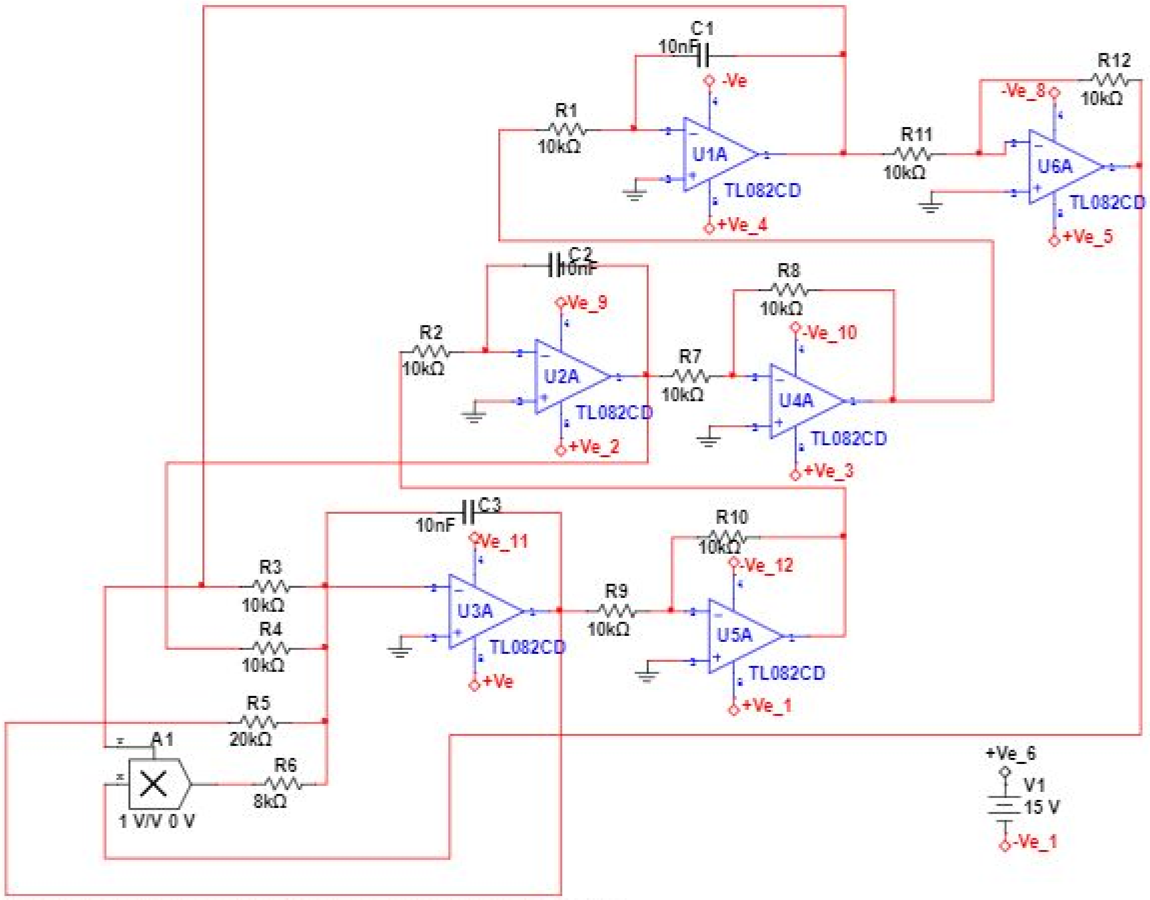


Figure 2: Electronic circuit design of Jerk chaotic system

The operational amplifiers use in this circuit are TL082CD and AD633JN multipliers of which the power supplies are $\pm 15volts$. We set the values of the circuital components as follow:

$$R_1 = R_2 = R_3 = R_4 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = 10k\Omega$$
$$R_5 = 20k\Omega, R_6 = 8k\Omega \quad (9)$$
$$C_1 = C_2 = C_3 = 10nF$$

By adopting the circuit designed technique, the results obtained in Figure 3 displayed the various attractors of the Jerk 3D chaotic system (1) when $a = 0.5$ and $b = 0.125$obtained in MultiSIM 14. Obviously the obtained oscilloscope results (Figure 3) confirmed the theoretical result (Figure 1).

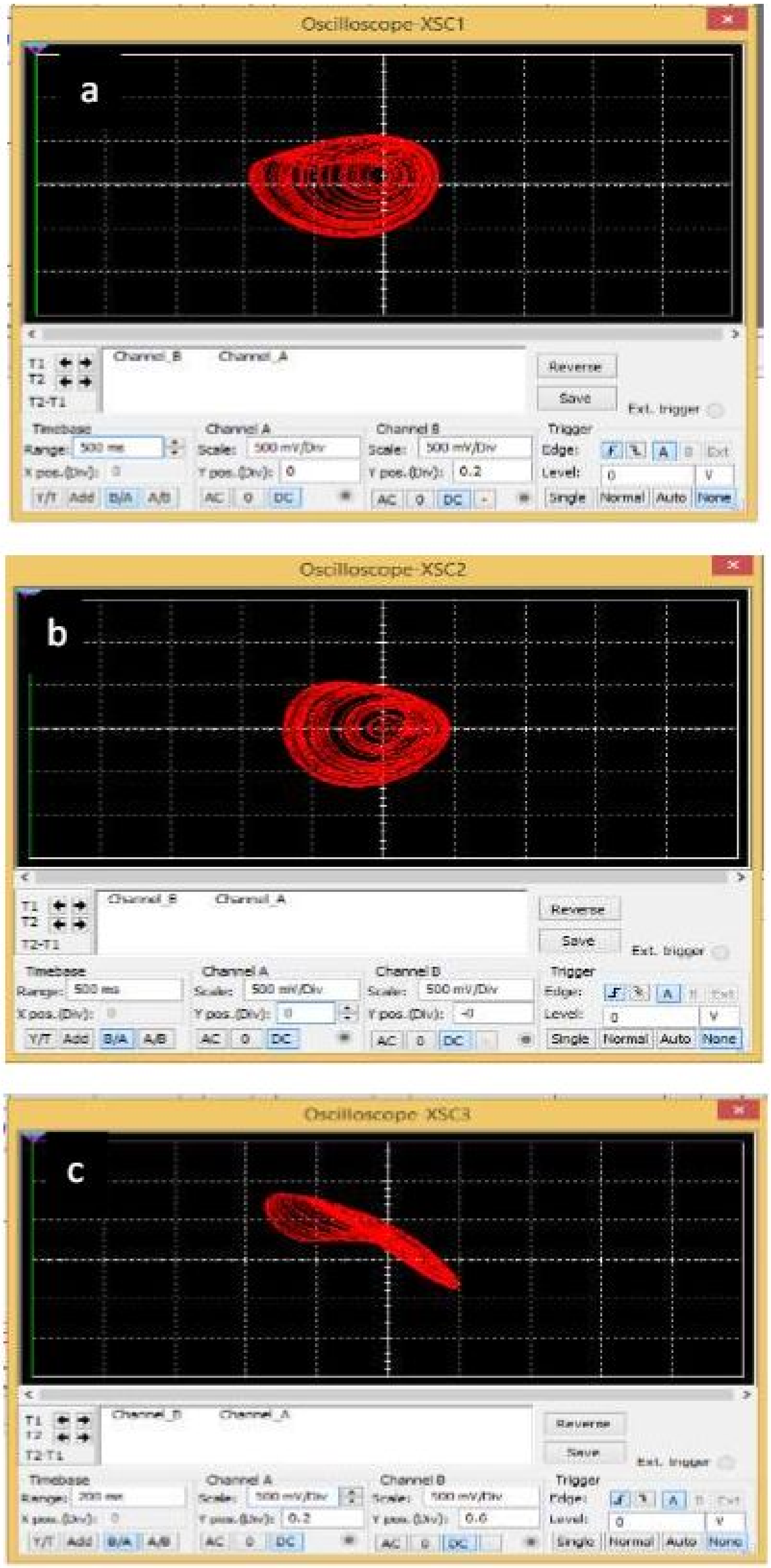


Figure 3: MultiSIM Phase space attractors of Jerk chaotic system (1); (a) $x - y$ plane, (b) $y - z$ plane and (c) $x - z$ plane

Robust Control

Recursive Backstepping Tracking Control

Here, the robust control is demonstrated via recursive backstepping technique to control the state variables x, y and $z$of the Jerk 3D chaotic system (1) to track a desire smooth function $r(t)$ at any chosen position.

The first step is to design a controller $u_i(t)$ $(i = x, y, z)$ by adding the control function $u_i(t)$ to equation (1).

$$\dot{x} = y + u_x$$
$$\dot{y} = z + u_y \quad (10)$$
$$\dot{z} = -x - y - az - bx^2 + u_z$$

The desire value of the state variables x, y and $z$are chosen to be x^*, y^* and z^* in order.

The error dynamics between the state variables (x, y and$z$) and the desire values is illustrated in equation (11):

$$e_x = x - x^*$$
$$e_y = y - y^* \quad (11)$$
$$e_z = z - z^*$$

In order to design a general control laws $u_i(t)$ ($i = x, y$and$z$) that can control equation (10) to track any trajectory $r(t)$ that is a smooth function of time, the desired values is chosen in equation (12) below.

$$x^* = r(t)$$
$$y^* = \alpha_1 e_x \quad (12)$$
$$z^* = \alpha_2 e_x + \alpha_3 e_y$$

where α_i $(i = 1,2,3)$ is the control scaling factor to be estimated.

The substitution of equation (12) into equation (11) and differentiating the result give the error dynamics in equation (13):

$$\dot{e}_x = e_y + \alpha_1 e_x - \dot{r}(t) + u_x$$
$$\dot{e}_y = e_z + \alpha_2 e_x + \alpha_3 e_y - \alpha_1 \dot{e}_x + u_y$$
$$\dot{e}_z = -\left(e_x + r(t)\right) - \left(e_y + \alpha_1 e_x\right) - a\left(e_z + \alpha_2 e_x + \alpha_3 e_y\right) - b(e_x + r(t))^2 - \alpha_2 \dot{e}_x - \alpha_3 \dot{e}_y + u_z \quad (13)$$

In order to stabilize the error dynamics system (13), the Lyapunov function is defined as follows;

$$V = \frac{1}{2}\left(e_x^2 + e_y^2 + e_z^2\right) \quad (14)$$

The time derivative of the equation (14) is:

$$\dot{V} = e_x \dot{e}_x + e_y \dot{e}_y + e_z \dot{e}_z \quad (15)$$

To satisfy the condition for asymptotic stability of the error vector (13) that is necessary for tracking, $\dot{V} = -\sum k i e_i^2 < 0$.

Therefore, the control function $u_i(t)$ $(i = x, y, z)$ is estimated from equation (13) as shown below.

$$u_x = -e_y - \alpha_1 e_x + \dot{r}(t) - k_x e_x$$
$$u_y = -e_z - \alpha_2 e_x - \alpha_3 e_y - k_y e_y$$
$$u_z = e_x + r(t) + e_y + \alpha_1 e_x + a(e_z + \alpha_2 e_x + \alpha_3 e_y) + b(e_x + r(t))^2 - k_z e_z \quad (16)$$

The substitution of equation (13) and (16) respectively into equation (15) yield;

$$\dot{V} = -k_x e_x^2 - k_y e_y^2 - k_z e_z^2 < 0 \quad (17)$$

At any equilibrium point, system (1) must have $y^* = z^* = 0$ (18)

Therefore, at the equilibrium point;

$$E_0 = 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (19)$$

And

$$\alpha_i = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (20)$$

These mean that;$y^* = z^* = 0$ from equation (12) which agreed with equation (18).

Hence, the control function in equation (16) reduces to;

$$u_x = -e_y + \dot{r}(t) - k_x e_x$$
$$u_y = -e_z - k_y e_y \quad (21)$$
$$u_z = e_x + r(t) + e_y + az + b(ex + r(t))^2 - k_z e_z$$

Numerical Simulation

To demonstrate the effectiveness and the feasibility of the proposed scheme, Runge-Kutta algorithm of fourth-order is used in the simulation with the initial conditions as follows; $(x, y, z) = (-1.0, 1.8, 3.0)$, a time step of$0.001$and the parameter values of a and b as in Figure 1 are fixed to ensure chaotic dynamics of the state variables. System (10) is solved with the controllers $u_i(t)$ (i = 1, 2, 3) as defined in (21) with the values of the control gain feedback constant as $k_x = 1.0$, $k_y = k_z = 8.0$.The results showed that the state variables move chaotically with time when the controller is deactivated and when the controller is switch on at time t = 50 the state variables are controlled to track the desired functions $r(t) = 2Cos0.8t$. From the results of the numerical simulations shown in Figure 4, we observed that the controllers (21) is capable of controlling the dynamics of the chaotic system (10) to track any desired smooth function, $r(t)$ and to stabilize it at any position P (case $r(t) = P$) and when P = 0 the system becomes stabilized at the origin.

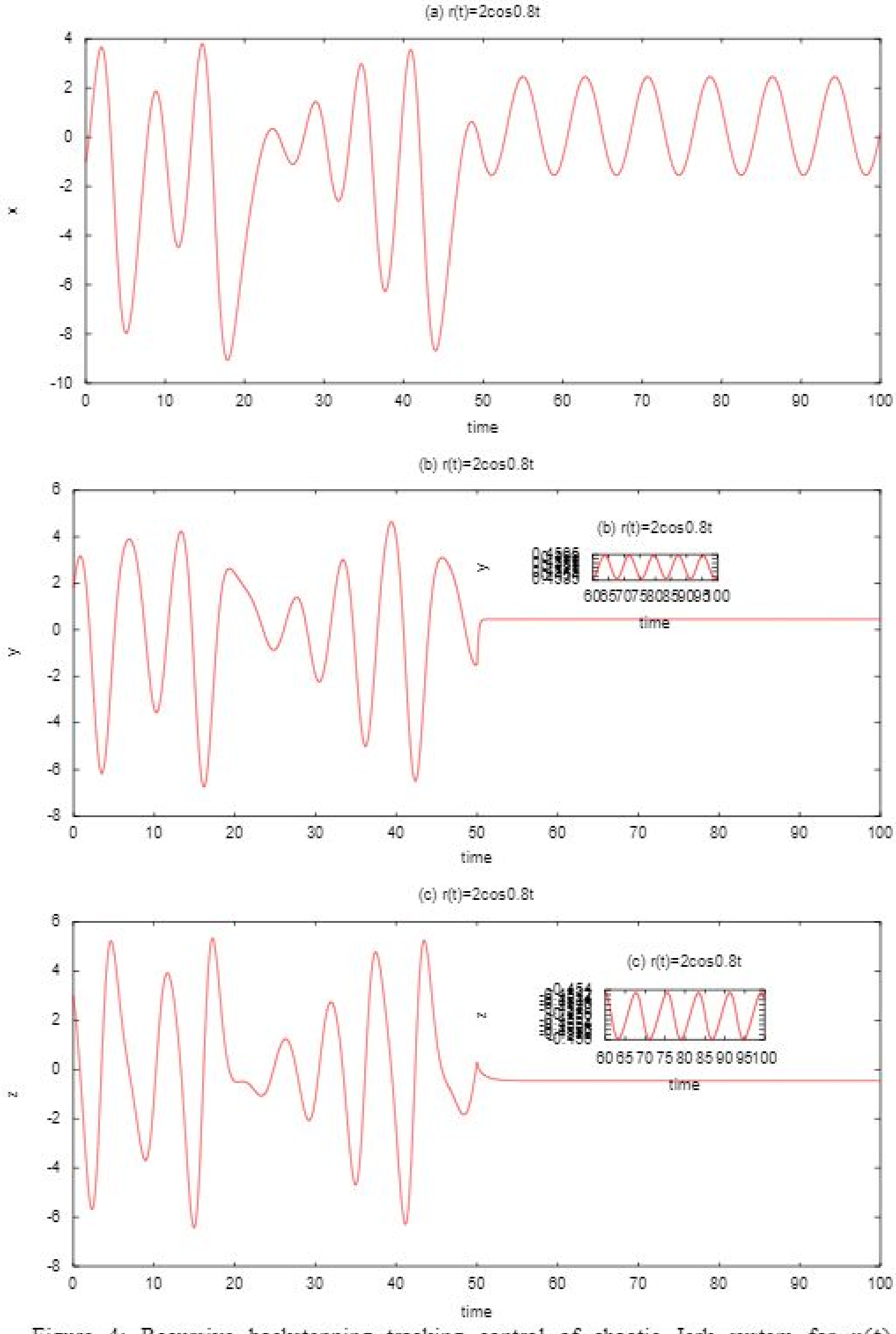


Figure 4: Recursive backstepping tracking control of chaotic Jerk system for $r(t) = 2Cos0.8t$ when the controller is activated at $t = 50$

Active backstepping Synchronization for two identical chaotic Jerk systems

To investigate the validity of the above method, a procedure via active backstepping technique as drive (master) and response (slave) to synchronize two identical Jerk chaotic systems was designed.

Design of the Active backstepping controller

From equation (1), we let$x = x_1$, $y = x_2$ and $z = x_3$ as follows;

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3 \quad (22)$$
$$\dot{x}_3 = -x_1 - x_2 - ax_3 - bx_1^2$$

The system (22) above is the drive (master) or transmitter and the following equation is the response (slave) or receiver.

$$\dot{y}_1 = y_2 + u_1(t)$$
$$\dot{y}_2 = y_3 + u_2(t) \quad (23)$$
$$\dot{y}_3 = -y_1 - y_2 - ay_3 - by_1^2 + u_3(t)$$

Where $u_i(t)(i = 1,2,3)$ are the nonlinear control functions to be determined for the synchronization of the systems (22) and (23).

The subtraction of equation (23) from equation (22) using the following notations $e_i = y_i - x_i$ or $y_i = e_i + x_i$ gives the error vector (24).

$$\dot{e}_1 = e_2 + u_1$$
$$\dot{e}_2 = e_3 + u_2 \quad (24)$$
$$\dot{e}_3 = -e_1 - e_2 - ae_3 - 2x_1e_1b - be_1^2 + u_3$$

The main objective here is to design the control function $u_i(t)$ to synchronize equations (22) and (23) as well as to stabilize the error vectors (24) at the origin at any choosing time.

The first equation in equation (24) is firstly stabilize by regarding e_2 as a controller and considering the quadratic Lyapunov function $V_1(e_1) = \frac{1}{2}e_1^2$ of which the time derivative gives equation (25).

$$\dot{V}_1 = e_1\dot{e}_2 = e_1[e_2 + u_1] \quad (25)$$

The virtual controller e_2 is estimated as; $e_2 = \alpha_1(e_1)$, then equation (25) becomes; $\dot{V}_1 = e_1[\alpha_1(e_1) + u_1]$, if $\alpha_1(e_1) = -e_1$ and $u_1 = 0$, then $\dot{V}_1 = -e_1^2$ negative definite.

The errorω_2 between e_2 and $\alpha_1(e_1)$ is defined as;

$$\omega_2 = e_2 - \alpha_1(e_1) \quad (26)$$
$$\therefore \omega_2 = e_2 - (-e_1) = e_2 + e_1 \quad (27)$$

The substitution of $\dot{e}_2$ and $\dot{e}_1$ from equation (24) above into the time derivative of equation (27) yields equation (28) below.

$$\dot{\omega}_2 = e_3 + e_2 + u_2 \quad (28)$$

Then, we stabilize (e_1, ω_2) subsystem given in equation (24) by chosen the Lyapunov function$V_2(e_1, \omega_2) = v_1(e_1) + \frac{1}{2}\omega_2^2$. Differentiating V_2 along the trajectory of error vector (24), gives equation (29).

$$\dot{V}_2(e_1, \omega_2) = \dot{v}_1(e_1) + \omega_2\dot{\omega}_2 \quad (29)$$

From equations (25) and (27),

$$\dot{v}_1(e_1) = -e_1^2 + \omega_2 e_1 \quad (30)$$
$$\therefore \dot{V}_2(e_1, \omega_2) = -e_1^2 + \omega_2 e_1 + \omega_2[e_3 + e_2 + u_2] \quad (31)$$

Substitution for e_2 from equation (27) in (31) gives; $\dot{V}_2 = -e_1^2 + \omega_2[e_3 + \omega_2 + u_2]$. If e_3 is estimated as $\alpha_2(e_1, \omega_2) = 0$ and $u_2 = -2\omega_2$, then $\dot{V}_2 = -e_1^2 - \omega_2^2$ (negative definite), (e_1, ω_2) subsystem is stabilize.

Finally, the complete system is stabilize by regarding ω_3 as the error between e_3 and $\alpha_2(e_1, \omega_2)$.

$$\omega_3 = e_3 - \alpha_2(e_1, \omega_2) = e_3 \quad (32)$$

Substituted $\dot{e}_3$ and e_3 from equations (24) and (32) respectively into the time derivative of equation (32) resulting in equation (33);

$$\dot{\omega}_3 = -\omega_2 - a\omega_3 - be_1(2x_1 + e_1) + u_3 \quad (33)$$

Choosing the Lyapunov function $V_3(e_1, \omega_2, \omega_3) = v_2(e_1, \omega_2, \omega_3) + \frac{1}{2}\omega_3^2$ and differentiating gives equation (34).

$$\dot{V}_3(e_1, \omega_2, \omega_3) = \dot{v}_2(e_1, \omega_2) + \omega_3\dot{\omega}_3 \quad (34)$$

Hence,

$$\dot{V}_3 = -e_1^2 - \omega_2^2 + \omega_3[-\omega_2 - a\omega_3 - be_1(2x_1 + e_1) + u_3] \quad (35)$$

If $u_3(t)$ is measurable as; $u_3 = be_1(2x_1 + e_1) + \omega_2$, then;

$\dot{V}_3 = -e_1^2 - \omega_2^2 - a\omega_3^2$ (Negative definite, since $a > 0$).

Clearly, the derivative of $\dot{V}_i$ $(i = 1,2,3)$ is negative definite, thus one can conclude that the synchronization error vector $e_i(t)$ in equation (24) is globally stable also the synchronization between systems (22) and (23) is achieved with the designed control function defined in equation (36).

$$u_1 = 0$$
$$u_2 = -2\omega_2 \quad (36)$$
$$u_3 = be_1(2x_1 + e_1) + \omega_2$$

Numerical Simulations for Designed Active backstepping controller

Using the Runge-Kutta fourth-order algorithm with the initial conditions of the drive and response systems$(x_1, x_2, x_3) = (0.1, 0.2, 0.1)$ and $(y_1, y_2, y_3) = (0.3, 0.1, -0.9)$ respectively with a time step of 0.001 and fixing the values of the parameters a and $b$as in Figure 1 to ensure complex chaotic dynamics of the state variables, systems (22) and (23) were solved with the control function defined in equation (36).The results of the simulation displays the states trajectories of the drive (master) and the response (slave) systems (22) and (23) in Figure 5 and the error dynamical system (24) in Figure 6 while Figure 7 give the norm of synchronization. The errors dynamic move chaotically with time when the controller is switch off and when the controller is switch on at $t = 50$ (Figure 6), the errors vector converges to zero, thereby guaranteeing the synchronization of systems (22) and (23) (Figure 6). This confirmed by the synchronization norm e given by

$$e = \sqrt{e_1^2 + e_2^2 + e_3^2} \quad (37)$$

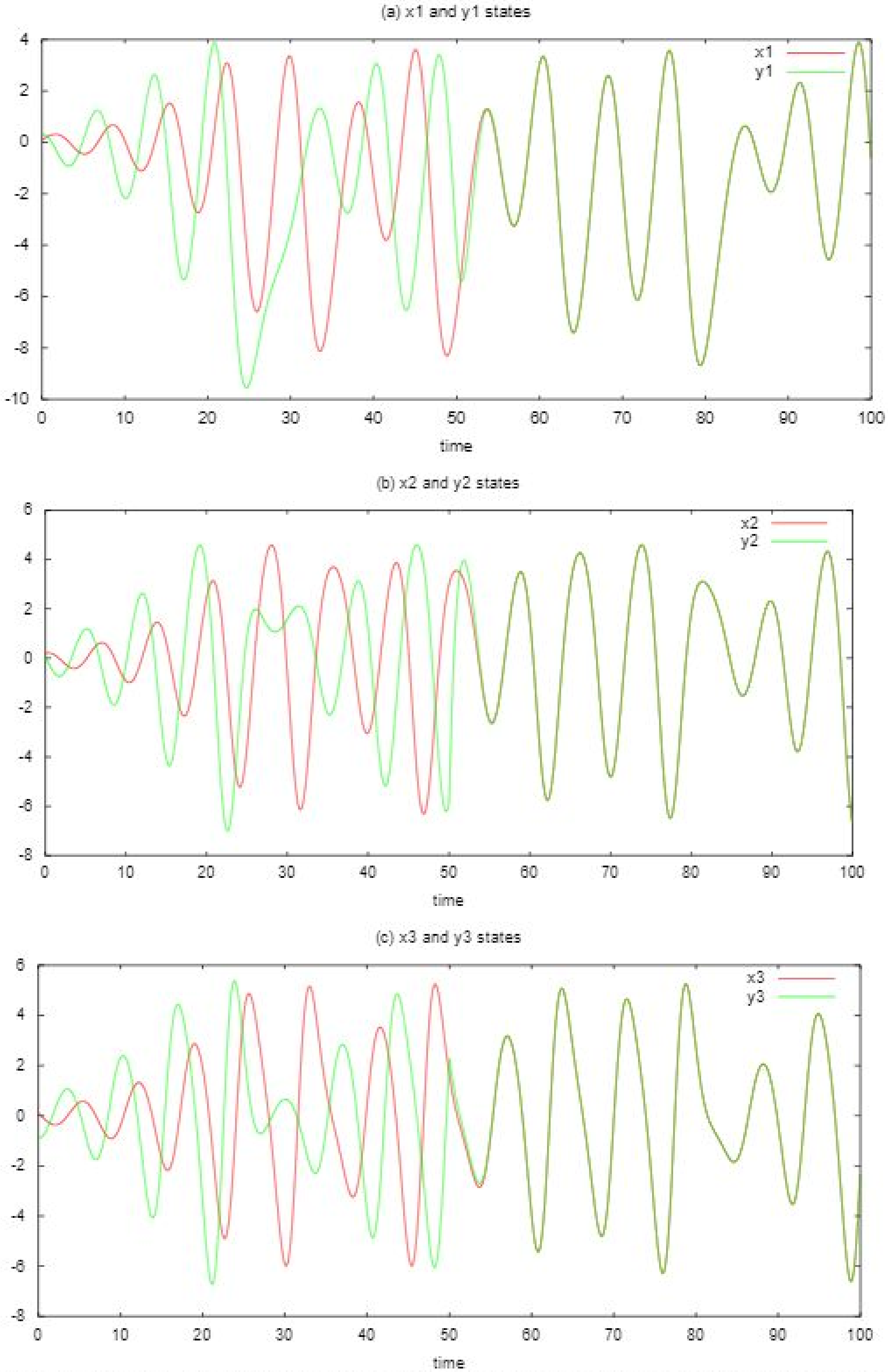


Figure 5: The time response of the state variables for master system (22) and the slave system (23) when the controller is activated at $t = 50$

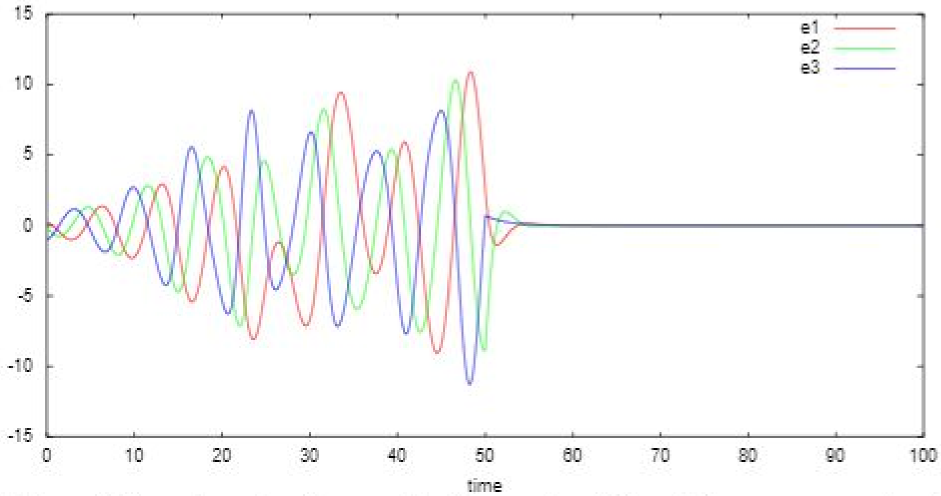


Figure 6: Error dynamics between the drive system (22) and the response system (23) when the controller is switched on at $t = 50$

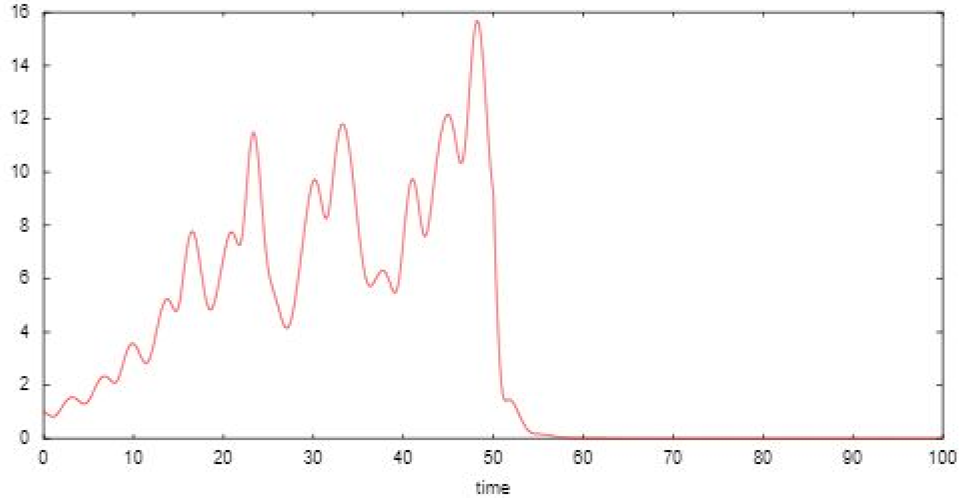


Figure 7: Synchronization Norm

Secure Communication

Cryptography is the science of protecting the privacy of information during transmission or saved for long time has assumes special attention in modern information system.

Chaotic cryptography is found applicable in image encryption, neural networking, secure communication and economics (Yau *et al.,* 2012; Laoye *et al.,* 2021, Adelaja *et al.,* 2021).

Figure 8 display analog communication scheme base on active backstepping synchronization in which four main components are included: information signal, chaotic encryption signal, chaotic decryption signal and decryption error signal.

In this method, we achieved the encryption by adding the information signal to the chaos wave signal carrier via additive routine. The information signal $i(t)$ is presented in equation (38).

$$i(t) = 3\,sin\,0.04t \qquad (38)$$

The encrypted information is masked with the chaos wave carrier x_i as shown in equation (39).

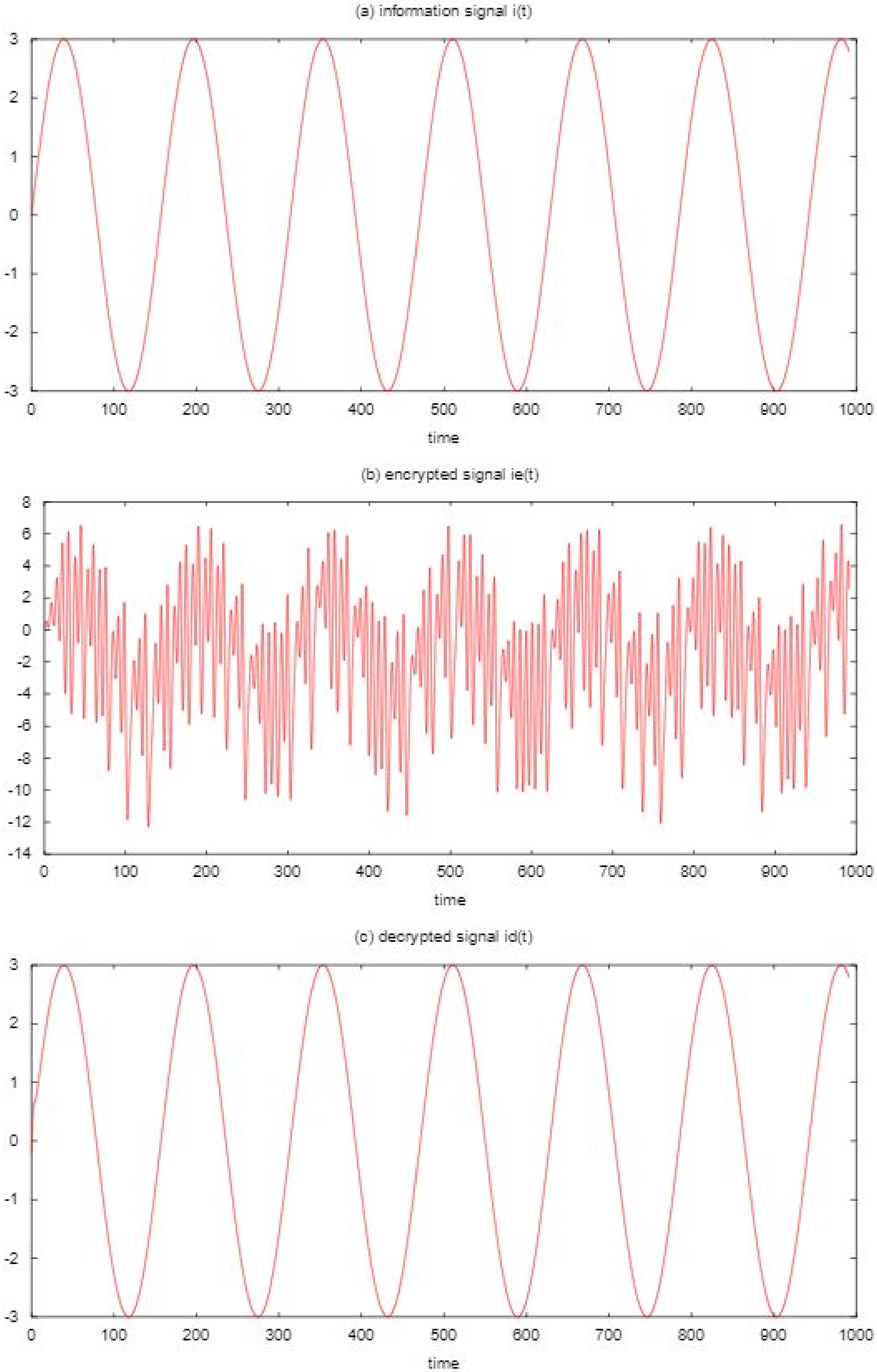
$$i_e(t) = i(t) + x_i \qquad (39)$$

The decrypted information $i_d(t)$ is extracted by the inverse function shown in equation (40).

$$i_d(t) = i_e(t) - y_i \qquad (40)$$

Hence, the chaos wave signal x_i of the master (drive) system is transmitted to the slave (response) system by coupling channel for synchronization between the drive system and the response system. The information signal $i(t) = 3\,sin\,0.04t$is remains masked in the encrypted signals $i_e(t)$ and transmitted to the receiver. The decrypted information $i_d(t)$ was later extracted by inverse function. Once the difference between the information signal $i(t)$ and the decrypted signal $id(t)$ approaches zero, it means that the information is recovered as shown in equation (41) below.

$$er = i(t) - id(t) \qquad (41)$$


(a) information signal i(t)
time
(b) encrypted signal ie(t)
time
(c) decrypted signal id(t)
time

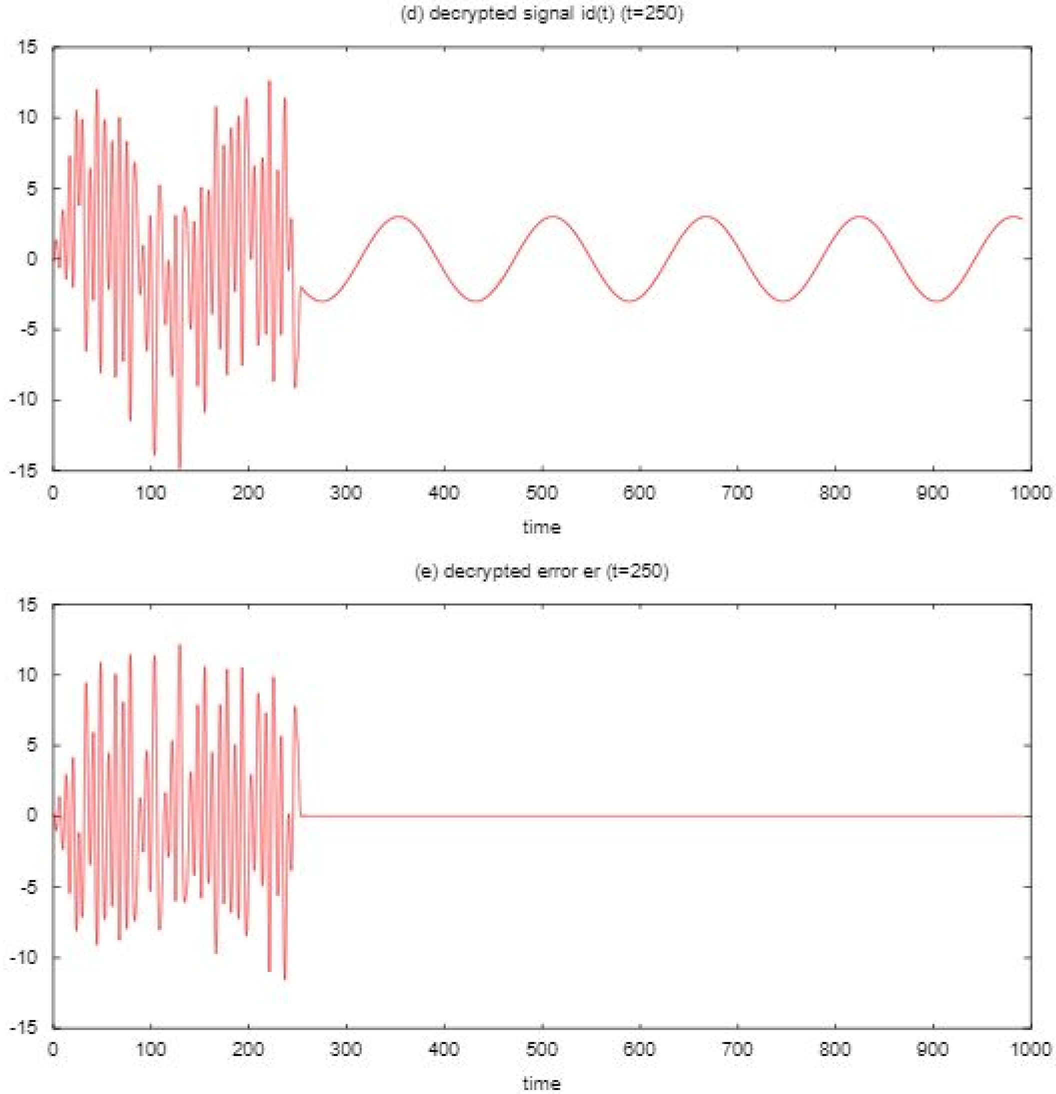


Figure 8: Jerk system masking communication; (a) information signal$i(t)$; (b) encrypted signal$i_e(t)$; (c) decrypted signal$i_d(t)$; (d) decrypted signal $i_d(t)$ (t=250)$i_d(t)$; (e) decrypted error $er = i(t) - id(t)$

CONCLUSION

This paper described the three-dimensional autonomous nonlinear Jerk chaotic system with one quadratic term and two parameters a and b. The system displays complex dynamical behavior as confirmed by the theoretical model and circuital implementation. The analytical results revealed that this system is dissipative with two dimensional unstable saddle points. As for engineering application, electronic circuit designed in Multism simulation was achieved. The results of the recursive backstepping tracking control and active backstepping synchronization were effective for tracking control and stabilization of the Jerk 3D chaotic system. It was investigated that chaos synchronization is equivalent to stabilizing the systems at the equilibrium point by determining a suitable feedback controller$u_i(t)$.The success of synchronization output were extended to secure communication. The numerical simulation results presented demonstrated the validity and the effectiveness of the proposed scheme. Hence, this scheme can be implemented in real applications such as cryptosystem, encryption, neural networks and secure communication.

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