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Exploring the Role of Binary Mass Ratio (q = 0.1, 1.0) in Shaping Gravitational Wave Amplitude, Frequency Evolution, and Waveform Morphology

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ABSTRACT

Compact binary mass ratio ($q = \frac{m_2}{m_1} = 0.1, 1.0 =$) lays a critical role in shaping the characteristics of gravitational waves (GWs) emitted during the inspiral, merger, and ringdown phases. This study employs numerical relativity (NR) simulations using the Einstein Toolkit to model binary black hole systems and examine the dependence of waveform amplitude, frequency evolution, and morphology on mass ratio. Equal-mass binaries ($q \approx 1$) produce symmetric, high-amplitude signals with smooth chirp behaviour, whereas increasingly asymmetric systems ($q = 0.1$) exhibit reduced amplitudes, precession-induced modulations, and enhanced excitation of higher-order modes. The results indicate amplitude reductions of up to 40% for $q = 0.1$ relative to $q = 1.0$, slower chirp rates, and waveform mismatches exceeding 1% in post-Newtonian models for strongly asymmetric configurations. These findings highlight the importance of accurately accounting for mass-ratio effects in gravitational-wave template banks, with direct implications for improved parameter estimation in LIGO–Virgo–KAGRA observations and future detectors such as LISA.

Keywords:
Gravitational waves,
Mass Ratio,
Waveform Morphology,
Compact Binaries.

INTRODUCTION

The direct detection of gravitational waves (GWs) from compact binary mergers opens a new observational window to strong-field gravity and relativistic astrophysics (Abbott et al., 2016). The observations from the Laser Interferometer Gravitational-Wave Observatory (LIGO) and the Virgo Collaboration have proven that GW waveforms contain information about the properties of the merging masses such as their masses, spins, and the dynamical history of the orbit (Phurailatpam et al., 2024). Related work has shown that gravitational-wave measurements can be used to study the innards of neutron stars using mass–radius relations and equation-of-state constraints through piecewise polytropic models (Bringen et al., 2025). One of these parameters is the binary mass ratio, $q = m_2/m_1$. This is particularly crucial for understanding the nature of the released gravitational radiation.

Waveforms from almost equal mass ($q \approx 1$) binary systems are strong and symmetric, while mass asymmetric ($q \ll 1$) systems cause large changes in amplitude, frequency evolution, and general morphology of waveforms (Mac Uilliam, Akcay & Thompson, 2024). These variances are due to asymmetries in the orbital dynamics, energy output and angular-momentum transport. Existing waveform models (e.g., TEOBResumS, IMRPhenomXPHM, SEOBNRv5PHM) are finding it more and more difficult to model systems with large mass ratios, high spins, or extreme asymmetries, with a number of models sometimes showing measurable mismatches with NR benchmarks (Mac Uilliam et al., 2024).

Astrophysical compact-binary systems span a broad range of mass ratios, from nearly symmetric binary black holes to neutron-star–black-hole candidates. However, detailed numerical relativity studies often focus on representative configurations rather than exhaustive parameter sweeps, particularly for strongly asymmetric systems. In this work, two representative cases are examined: an equal-mass binary ($q = 1.0$) and a highly asymmetric system ($q = 0.1$). This study investigates how these contrasting mass-ratio configurations influence gravitational-wave amplitude, frequency evolution during inspiral and merger, and waveform morphology, including phase evolution and harmonic content. The results are used to assess implications for gravitational-wave detection, parameter estimation, and template-bank development for current (LIGO/Virgo/KAGRA) and future observatories.

Theoretical and Numerical Framework

To accurately model gravitational waveforms from compact binary coalescences, one must have a clear understanding of the basic principles that govern gravitational radiation, as well as different theoretical approaches that cover the weak-field inspiral, strong-field merger, and linearised ringdown phases. In general relativity, gravitational waves emerge from time-varying mass distributions characterised by a non-vanishing quadrupole moment. The gravitational-wave strain detected at a distance D from the source is directly proportional to the second time derivative of the mass quadrupole tensorQ_{ij},

$$h(t) \propto \frac{1}{D}\ \frac{d^2Q_{ij}}{dt^2} \tag{1}$$

For compact binary systems, the evolution of the gravitational-wave phase and frequency during the inspiral is predominantly dictated by the chirp mass$\mathcal{M}$, which reflects the dependence on the component masses,

$$\mathcal{M} = \frac{(m_1m_2)^{\frac{3}{5}}}{(m_1+m_2)^{\frac{1}{5}}} \tag{2}$$

Where$m_1 \geq m_2$. If the total mass stays the same, increasing mass asymmetry lowers the chirp mass. This makes the frequency change more slowly and makes it more sensitive to higher-order corrections. These basic connections make it necessary to use post-Newtonian, effective-one-body, and numerical relativity methods to model the entire inspiral–merger–ringdown waveform.

Post-Newtonian (PN) theory systematically expands Einstein's field equations in powers of $(v/c)^n$, where v denotes the characteristic orbital velocity, and remains applicable in the weak-field, adiabatic inspiral regime. For orbits that are almost circular, the quadrupole formula gives the leading-order gravitational-wave luminosity:

$$\mathcal{F}_{Newt} = \frac{32}{5}\frac{c^5}{G}\ \nu^2x^5 \tag{3}$$

Where $x = (GM\omega/c^3)^{2/3}$ defines the PN expansion parameter, $\nu = \mu/M = q/(1+q)^2$ is the symmetric mass ratio, and ω denotes the orbital frequency. Higher-order corrections, extending through fourth post-Newtonian (4PN) order, incorporate tail effects, spin–orbit couplings at 1.5PN order, and nonlinear memory contributions,

$$\mathcal{F} = \mathcal{F}_{\mathrm{Newt}}[1 + \textstyle\sum_{k=2}^{\infty}\ a_kx^k] \tag{4}$$

With coefficients a_k depending explicitly on the mass ratio and spin projections (Blanchet, 2014; Blanchet et al., 2022).

PN corrections are also included in the two-body relative acceleration,

$$\ddot{\mathbf{r}} = -\frac{GM}{r^2}\hat{r} + \frac{GM}{c^2r^2}[A(v)\hat{r} + B(v)(\hat{n}\cdot\hat{r})\hat{n}] + \mathcal{O}(c^{-4}) \tag{5}$$

Where the velocity-dependent functions A and B scale with the symmetric mass ratio$\nu(q)$. As the system nears merger$(v \rightarrow c)$, PN approximations increasingly fail to maintain precision, resulting in residual waveform discrepancies $h_{\mathrm{PN}} - h_{\mathrm{NR}} \sim O(v^5)$ that may skew parameter estimation based on the Fisher matrix.

Effective-One-Body (EOB) theory overcomes these constraints by translating two-body dynamics into geodesic motion within an effective spacetime, utilising free parameters adjusted against numerical relativity (NR) simulations to guarantee continuity throughout inspiral, merger, and ringdown (Buonanno & Damour, 1999). For mass ratios$q\ >\ 0.25$, hybrid PN–EOB–NR waveform models like SEOBNRv5 and TEOBResumS can get mismatches of less than 1%. However, they don't work as well when the mass asymmetries are more extreme.

Numerical relativity offers direct, approximation-free solutions to Einstein's field equations in the strong-field regime, establishing it as a crucial benchmark for waveform modelling (Pretorius, 2005; Campanelli et al., 2006). The Einstein Toolkit (Löffler et al., 2012) was used to run the simulations in this study. The McLachlan thorn was used to implement spacetime evolution using the Baumgarte-Shapiro-Shibata-Nakamura (BSSN) formulation. The 3+1 Arnowitt–Deser–Misner (ADM) decomposition of the spacetime metric is given by

$$ds^2 = -\alpha^2dt^2 + \gamma_{ij}(dx^i + \beta^idt)(dx^j + \beta^jdt) \tag{6}$$

Where α is the lapse function, β^i the shift vector, and γ_{ij} the spatial metric. The extrinsic curvature evolves according to

$$K_{ij} = -\frac{1}{2}(\partial_t\gamma_{ij} - D_i\beta_j - D_j\beta_i) \tag{7}$$

Moving-puncture gauge conditions make sure that the evolution stays stable through merger, with,

$$\partial_t\alpha = -2\alpha K,$$
$$\partial_t\beta^i = \frac{3}{4}B^i,$$
$$\partial_tB^i = \partial_t\beta^i - \eta B^i \tag{8}$$

The computational domain used Carpet adaptive mesh refinement with eight levels of refinement. It was able to dynamically track the black hole horizons and had a finest grid spacing of $\Delta x\ =\ 0.02M$. The TwoPunctures thorn was used to make the first data. It was assumed that the orbits were almost circular and had very little eccentricity $(e_0{\approx}10^{-3})$ and that the initial separations$r_0 =\ 6 - 12M)$. To reduce false reflections, Sommerfeld-type radiative boundary conditions were used, and to reduce high-frequency numerical noise, Kreiss–Oliger dissipation was used.

Gravitational radiation was extracted using the Weyl scalar $\psi_4 = \partial_th_+ - i\partial_th_\times$ at extraction radii $R_{\mathrm{ext}} = 40 - 100M$ and decomposed into spin-weighted spherical harmonics,

$$h(t,r) = \textstyle\sum_{l,m}h_{lm}(t,r)\ {}_{-2}Y_{lm}(\iota,\phi) \tag{9}$$

With subsequent processing conducted through the Kuibit analysis framework (Bozzola, 2021).

The primary $(l = 2, m = 2)$ mode delineates nearly equal-mass systems$(q \approx 1)$, whereas higher-order modes gain prominence for (q < 0.5), inducing observable alterations in waveform morphology.

We used the dephased mismatch over the LIGO sensitivity band to measure waveform fidelity,

$$\mathcal{M} = 1 - \max_{t_c,\phi_c} \frac{|\langle h_1|h_2\rangle|}{\sqrt{\langle h_1|h_1\rangle\langle h_2|h_2\rangle}} \qquad (10)$$

Where the noise-weighted inner product is defined as

$$\langle a \mid b\rangle = 4\Re\int_{f_{\text{low}}}^{f_{\text{high}}} \frac{\tilde{a}(f)\tilde{b}^{*}(f)}{S_n(f)}\,df \qquad (11)$$

The integration limits were set to f_{low} = 20 Hz and f_{high} = 1024 Hz, with a target mismatch threshold of $\mathcal{M} < 1\%$, consistent with current LIGO sensitivity requirements (LIGO Scientific Collaboration, 2010).

MATERIALS AND METHODS

Compact binary black hole systems were modelled using two representative mass-ratio configurations: an equal-mass system ($q = m_2/m_1 = 1.0$) and a highly asymmetric system ($q = 0.1$). These correspond to total masses of $M = 60\, M_\odot$ and $M = 110\, M_\odot$, respectively, consistent with stellar-mass binary black hole systems observed by the LIGO–Virgo Collaboration. All simulations were performed using the Einstein Toolkit numerical relativity framework, with waveform post-processing and analysis carried out using Python-based tools (Bozzola, 2021).

The spacetime evolution was handled using the Baumgarte–Shapiro–Shibata–Nakamura (BSSN) formulation as implemented in the McLachlan thorn, which solves Einstein's field equations within a stable 3+1 decomposition. Initial data were generated using the TwoPunctures thorn under moving-puncture conditions, assuming quasi-circular orbits. Adaptive mesh refinement was implemented with the Carpet infrastructure using eight refinement levels, achieving a finest grid resolution of $\Delta x = 0.02M$ and dynamically tracking the black hole horizons throughout the inspiral, merger, and ringdown phases.

Gauge conditions consisted of the standard 1+log slicing condition for the lapse function and the Gamma-driver condition for the shift vector. Sommerfeld-type radiative boundary conditions were applied at the outer boundaries to minimise unphysical reflections, while Kreiss–Oliger dissipation was employed to suppress high-frequency numerical instabilities.

Gravitational radiation was extracted in terms of the Weyl scalar ψ_4 at multiple coordinate radii in the range $R = 40$–$100M$. The extracted waveforms were decomposed into spin-weighted spherical harmonic modes and post-processed using the Kuibit analysis framework for waveform reconstruction, visualisation, and quantitative comparison.

Parameter Space

We perform numerical simulations for representative configurations of stellar-mass binary black hole systems in order to isolate the effect of the mass ratio asymmetry on the gravitational wave emission. Rather than performing a full parameter sweep, this study focuses on two contrasting cases: an equal-mass binary ($q = 1.0$) and a highly asymmetric system ($q = 0.1$).

The equal-mass system consists of component masses $m_1 = m_2 = 30\, M_\odot$, corresponding to a total mass of $60\, M_\odot$, while the asymmetric system is defined by $m_1 = 100\, M_\odot$ and $m_2 = 10\, M_\odot$, giving a total mass of $110\, M_\odot$. These configurations provide a direct comparison between symmetric and strongly asymmetric binaries within the mass range observed by current gravitational-wave detectors.

Initial conditions assume quasi-circular orbits with negligible eccentricity ($e_0 \approx 0$), and initial separations in the range $r_0 = 6$–$12M$. Gravitational-wave extraction is performed at multiple radii $R_{\text{ext}} = 40$–$100M$ to ensure robustness of the waveform signals. The parameters used in the simulations are summarised in Table 1.

Table 1: Parameter Space Used for Numerical Simulations

Parameter	Symbol	Range/Values	Units
Primary BH mass	m_1	30,100	$M_\odot$
Secondary BH mass	m_2	30, 10	$M_\odot$
Mass ratio	$q = m_2/m_1$	0.1, 1.0	-
Total mass	$\mathrm{M} = m_1 + m_2$	60, 110	$M_\odot$
Initial separation	r_0	6M, 12M	M
Eccentricity	e_0	0	-
Extraction radius	R_{ext}	40M, 100M	M

Waveform Validation and Model Comparison

To evaluate the stability and physical accuracy of the numerical relativity (NR) waveforms produced in this research, a validation procedure was undertaken, involving a comparison with established semi-analytical waveform models frequently employed in gravitational-wave data analysis. These models encompass post-Newtonian (PN) approximants, which are relevant in the initial inspiral phase, and effective-one-body (EOB) and phenomenological inspiral–merger–ringdown (IMR) models, all of which are calibrated using numerical relativity simulations (Buonanno & Damour, 1999; Pan et al., 2011; Varma et al., 2019).

Such comparisons are essential, as current LIGO–Virgo–KAGRA searches rely heavily on these models for matched-filter detection and parameter estimation across a wide range of source parameters (Abbott et al., 2019).

The validation focused on the dominant (ℓ, m) = (2,2) mode of the gravitational-wave strain, which carries the majority of the radiated energy and is most relevant for detection. For each simulated mass ratio, NR waveforms were aligned in time and phase with corresponding analytical waveforms during the early inspiral, where PN theory is expected to be accurate. Qualitative agreement was observed in both amplitude and phase evolution at early times, confirming the consistency of the numerical simulations with analytical expectations (Blanchet, 2014).

As the system's evolution progresses toward coalescence, the disparities among waveform models become more pronounced, especially in the context of mass-asymmetric binaries. For systems with nearly equal masses ($q \gtrsim 0.5$), numerical relativity (NR) waveforms exhibit strong concordance with effective-one-body (EOB) and inspiral-merger-ringdown (IMR) models throughout the inspiral and merger stages, with only minor phase discrepancies observed near the peak amplitude. Conversely, for systems with increasing mass asymmetry ($q \lesssim 0.25$), significant discrepancies arise during the late inspiral and merger, reflecting the increasing impact of higher-order modes and nonlinear dynamics, which are more difficult to accurately model within semi-analytical frameworks (Pan et al., 2011; Varma et al., 2019).

These differences are primarily seen as phase shifts and changes in amplitude in the later part of the waveform. This is consistent with increased mode mixing and precession in binary systems where the masses are not equal. Although current IMR models include higher harmonics, their accuracy decreases when the mass ratio is very different from one. This highlights the importance of specific numerical relativity simulations in this area of parameter space. The results of this study support earlier findings that waveform modeling uncertainties increase with mass asymmetry. If these uncertainties are not properly considered, they can cause systematic errors in parameter estimation (Abbott et al., 2019).

Generally, this validation confirms that the numerical waveforms used in this research are physically consistent with established analytical models in their respective domains of applicability, while also highlighting the limitations of current semi-analytical approaches for strongly asymmetric binaries. These findings reinforce the motivation for the present investigation and demonstrate the value of numerical relativity in accurately capturing waveform morphology across a broad mass-ratio range.

RESULTS AND DISCUSSION

Figure 1 illustrates the late inspiral and merger phases for both configurations. The left panel shows the gravitational-wave strain h_+ as a function of time, comparing the equal-mass system ($30M_{\odot} + 30\,M_{\odot}$, $q = 1.0$, solid blue line) with the asymmetric system ($100\,M_{\odot} + 10\,M_{\odot}$, $q = 0.1$, dashed red line). The right panel presents the corresponding instantaneous gravitational-wave frequency evolution for the same systems, with the merger time aligned at$t = 0$.

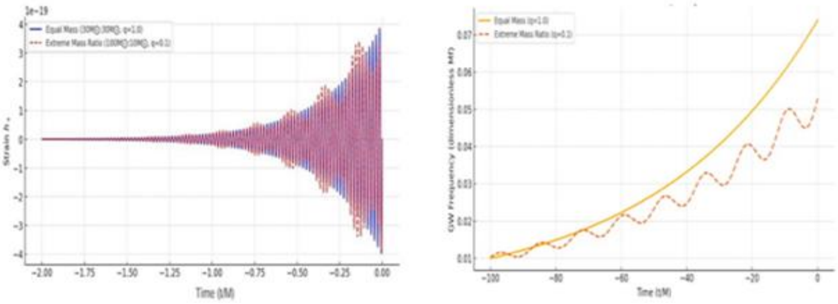

Figure 1: Gravitational-Wave Strain and Frequency Evolution for Binary Black Hole Systems with Different Mass Ratios

The waveform of the asymmetric system ($q = 0.1$) exhibits pronounced amplitude modulations, appearing as a periodic “beating” pattern that reflects enhanced precessional dynamics and asymmetry in orbital motion (Apostolatos et al., 1994). This system also exhibits higher frequency oscillatory features than the equal mass case. The $q = 1.0$ waveform, on the other hand, increases smoothly in amplitude with a monotonic chirp and exhibits little modulation, as expected for symmetric binaries.

The frequency evolution further highlights the contrast between the two configurations. The asymmetric system shows quasi-periodic variations in frequency associated with modulated orbital dynamics, while the equal-mass system follows a steady and continuous increase characteristic of non-precessing inspiral. The separation between the frequency curves reflects differences in total mass and dynamical evolution.

A clear dependence of gravitational-wave amplitude on mass ratio is observed. The equal-mass system produces the strongest signal, while the asymmetric system shows a reduction in amplitude of up to 40%. This reduction arises from the decreased efficiency of quadrupolar radiation in asymmetric mass configurations. Such attenuation may reduce detectability at larger luminosity distances.

We find that the waveform structure changes drastically with increasing mass asymmetry, from a smooth signal dominated by the quadrupole to a more complex modulated pattern. The numerical results are consistent with theoretical expectations and confirm that the mass ratio is a primary factor in determining the characteristics of the gravitational-wave signal.

CONCLUSION

This work emphasizes the important role of the binary mass ratio on the amplitude, frequency evolution, and morphology of gravitational-wave signals emitted during compact binary coalescences. Numerical relativity simulations of two representative configurations, an equal-mass system ($q = 1.0$) and a highly asymmetric system ($q = 0.1$), reveal that the asymmetric case shows an amplitude suppression of up to 40% and a significantly slower chirp than the equal-mass binary. Furthermore, the asymmetric system shows strong amplitude modulations and enhanced higher-order mode features while the equal-mass configuration leads to a smoother quadrupole-dominated waveform.

These findings highlight limitations in current gravitational-wave modelling, as post-Newtonian and effective-one-body waveform models can show mismatches exceeding $\mathcal{M} > 2\%$ when applied to strongly asymmetric systems, with potential consequences for parameter estimation. The modulation patterns and reduced inspiral rates identified here provide useful diagnostics for distinguishing mass-ratio effects from other physical influences in observed signals. For current LIGO-Virgo-KAGRA observations, the results motivate improved coverage of asymmetric binaries in template construction, while future detectors such as LISA are expected to benefit from more accurate modelling of systems with pronounced mass asymmetry.

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